

TWO FLUID COSMOLOGICAL MODEL IN BIANCHI V_0 IN SCALE COVARIANT THEORY OF GRAVITATION

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ABSTRACT

The present paper deals with Bianchi type V two fluids cosmological model in scale Covariant theory of gravitation. Matter fluid modeling observed matter and radiating Fluid modeling cosmic microwave background radiation taken as source. Exact solutions Of the field equations are obtained. Both interacting and non-interacting cases of two fluids are investigated. The exact solutions are obtained for constraints $X_1 = X_2 = X_3 = 0$. The Energy densities are positive for positive value of parametric constant α in case of exponential model. Energy transfer from matter to radiation is observed in case of interacting fluid. Some physical parameter of the obtained model is discussed in detail.

Keywords: Two fluid, Bianchi type V_0 , scale covariant theory.

1. INTRODUCTION

Cosmology is the study of the universe as a whole. The general theory of relativity provides basic tools for constructing cosmological models of the universe. It is generally acclaimed as a mathematically precise and physically sound theory of gravitation. However, in recent years, there has been a lot of interest in several alternative theories of gravitation. Brans-Dicke (BD) theory [1] is one of the noteworthy among the various modification of general relativity. BD theory introduces a dynamical scalar field to account for variable gravitational constant G . Nordtvedt [2] proposed a general class of scalar-tensor theories in which the parameter w of the BD theory is allowed to be an arbitrary function of the scalar field. In Saez-Ballester's theory [3] metric is coupled with dimensionless scalar field. Like BD theory, there is another viable alternative to general relativity which admits a variable G proposed by Canuto et al.[4]. The cosmological constant appears as a variable parameter in the framework of scale covariant theory. In scale covariant theory, Einstein's field equations are valid in gravitational units, whereas physical quantities are measured in the atomic units. The metric tensors in the two systems of units are related by a conformal transformation, $\bar{g}_{ij} = \phi^2(x^k) g_{ij}$ (1) where a bar denotes gravitational units and unbarred denotes atomic units. An important feature of this theory is that no independent equation for ϕ exists. Beesham [5], Venkateswarlu [6], Reddy et al.[7], Ram et al.[8], Zeyanddin and Saha [9], Katore et al.[10] are some of the authors who have investigated several aspects of the scale covariant theory of gravitation. Two fluid models, including energy densities of radiation and matter, are cosmologically important. Cosmological observations suggest that the radiation frame and the matter frame of the universe may not coincide [11]. The radiating fluid is modeling cosmic microwave background. The matter-fluid modeling the observed matter content of the universe. Recently, researchers have been taking keen interest in two fluids cosmological models. Amirhashchi et al.[12] have evaluated interacting two-fluid dark energy models in a non-flat universe. Khalatnikov et al. [13] have studied the quasi-isotropic expansion for a simple two-fluid cosmological models, including radiation and string gas. Coley and Dunn [14] have investigated the two fluids source of Bianchi type VI_0 models. Pant and Oli [15] have examined the Bianchi type II space-time with a two-fluid cosmological model. The paper is organized as follows: section 2 contains metric and field equations. Section 3 is devoted to solutions for non-interacting cases of fluids. In section 4, we present solutions in case of fluid interaction. In section 5, we conclude our obtained results.

2. Metric and field Equations:

We consider the Binchi type V_0 metric in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2mx} dy^2 - C^2 e^{-2mx} dz^2 \quad (2)$$

Where the metric functions A,B,C functions of t only and m is constant.

The field equation in scale covariant theory is given by

$$R_i^j - \frac{1}{2} R_j^i + f_j^i(\phi) = -8\pi G T_j^i \quad (3)$$

Where, ϕ is function of t only

$$\phi^2 f_{ij} = 2\phi\phi_{;ij} - 4\phi_{;i}\phi_{;j} - \phi_{ij}(2\phi\phi_{;k}^k - \phi'^k\phi_{,k}) \quad (4)$$

Here, ϕ is the scalar function of t only and other symbols have their usual meanings as in Riemannian geometry. The energy momentum tensor for two fluid is given by

$$T_{ij} = (T^m)_{ij} + (T^r)_{ij} \quad (5)$$

Where, $(T^m)_{ij}$ is the energy momentum tensor for matter field and $(T^r)_{ij}$ radiation field which is given by

$$(T^m)_{ij} = (p_m + \rho_m) u_i^m u_j^m - p_m g_{ij} \quad (6)$$

$$(T^r)_{ij} = \frac{4}{3} \rho_r u_i^r u_j^r - \frac{1}{2} \rho_r g_{ij} \quad (7)$$

From the CMB is interacting phases matter obeys the equation of state. The interactive Phase describes the pre-recombination era where the photons were bound to the matter(21)

Where, ρ_m is the energy density of matter, p_m is the pressure of the matter and ρ_r is the energy density of radiation with

$$g^{ij} u_i^m u_j^m = 1 \quad \text{and} \quad g^{ij} u_i^r u_j^r = 1 \quad (8)$$

We assume that the matter and radiation are both comoving which imply that $(T^m)_{ij}$

$$u_i^m = (0,0,0,1), u_i^r = (0,0,0,1)$$

Using equation (1),(2),(3) and (8) the field equations of the scale covariant can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{\phi}}{A\phi} - \frac{\dot{B}\dot{\phi}}{B\phi} - \frac{\dot{C}\dot{\phi}}{C\phi} - \frac{\ddot{\phi}}{\phi} + \left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{m^2}{A^2} = -8\pi G(p_m + \frac{1}{3}\rho_r) \quad (9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{\dot{B}\dot{\phi}}{B\phi} - \frac{\dot{C}\dot{\phi}}{C\phi} - \frac{\ddot{\phi}}{\phi} + \left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{m^2}{A^2} = -8\pi G(p_m + \frac{1}{3}\rho_r) \quad (10)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{\phi}}{A\phi} - \frac{\dot{B}\dot{\phi}}{B\phi} + \frac{\dot{C}\dot{\phi}}{C\phi} - \frac{\ddot{\phi}}{\phi} + \left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{m^2}{A^2} = -8\pi G(p_m + \frac{1}{3}\rho_r) \quad (11)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{\phi}}{A\phi} - \frac{\dot{B}\dot{\phi}}{B\phi} - \frac{\dot{C}\dot{\phi}}{C\phi} + \frac{\ddot{\phi}}{\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 - 3\frac{m^2}{A^2} = 8\pi G(\rho_m + \rho_r) \quad (12)$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 2\frac{\dot{A}}{A} \quad (13)$$

$$A^2 = BC \quad (14)$$

Where, the overhead dot denotes differentiation with respect to t

3. Metric –Interacting Model: -

Using equations(9) and (10) we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) \frac{\dot{C}}{C} + 2\left(\frac{\dot{B}}{B}\right) - \left(\frac{\dot{A}}{A}\right) \frac{\dot{\phi}}{\phi} = 0 \quad (15)$$

Equation(13) further reduces to

$$\frac{d}{dt} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) - 2 \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = 0 \quad (16)$$

$$\text{Let } V = a^3 \quad (17)$$

Making the use of equation(15) in equation(14), we yield

$$\frac{d}{dt} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) \left(2 \frac{\dot{\phi}}{\phi} - \frac{\dot{V}}{V} \right) \quad (18)$$

Integrating equation (16) , we get

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = \frac{X \phi^2}{V} \quad (19)$$

Equation (17) further reduce to

$$\frac{A}{B} = d_1 \exp \left(x_1 \int \frac{\phi^2}{V} dt \right) \quad (20)$$

$$\frac{A}{C} = d_2 \exp \left(x_2 \int \frac{\phi^2}{V} dt \right) \quad (21)$$

$$\frac{B}{C} = d_3 \exp \left(x_3 \int \frac{\phi^2}{V} dt \right) \quad (22)$$

Where d_i ($i=1,2,3$) and x_i ($i=1,2,3$) satisfy the relation $d_1, d_2, d_3, x_1, x_2, x_3$ are constant which satisfy the condition.

By using equation (14) and in equation (20),(21),(22) we get

$$A = V^{\frac{1}{3}} \quad (23)$$

$$B = D V^{\frac{1}{3}} \exp \left(X \int \frac{\phi^2}{V} dt \right) \quad (24)$$

$$C = D V^{\frac{1}{3}} \left(X \int \frac{\phi^2}{V} dt \right) \quad (25)$$

Where D,X are integration constant.

$$\text{By power law, here we take } V = a t^b \quad (26)$$

a,b are constant.

The gravitational term G is to be time dependent. We establish that G is decreasing of time

However the possibility of increasing the function of time cannot be neglected. We assume the most simple and useful form of G as

$$G = \alpha t \quad (27)$$

Where, α is the proportionality constant.

$$\text{We consider the scale function as } \phi(t) = \left(\frac{t_0}{t}\right)^\epsilon, \epsilon = \pm 1, \pm \frac{1}{2} \quad (28)$$

Where t_0 is constant.

$$A = a^{\frac{1}{3}} b^{\frac{1}{3}} \quad (29)$$

$$B = D a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left(\frac{X}{a} \frac{t^{-b+3}}{-b+3}\right) \quad (30)$$

$$C = D a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left(-\frac{X}{a} \frac{t^{-b+3}}{(-b+3)}\right) \quad (31)$$

$$ds^2 = dt^2 - a^{\frac{1}{3}} b^{\frac{1}{3}} dx^2 - D a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left(\frac{X}{a} \frac{t^{-b+3}}{-b+3}\right) e^{-2mx} dy^2 - D a^{\frac{1}{3}} t^{\frac{b}{3}} \exp\left(-\frac{X}{a} \frac{t^{-b+3}}{(-b+3)}\right) e^{-2mx} dz^2 \quad (32)$$

3. Physical and Kinematical properties :

We assume the relation between pressure and density of matter find through the “gamma-law” equation of state which is given by

3.1 Matter Density:-

$$\rho_m = \frac{2b^2-3b-2bt}{6\pi\alpha(4-3\gamma)t^3} + \frac{4\exp t^{-b+2}(b-3t)}{12\pi\alpha(4-3\gamma)t^2} - \frac{3m^2}{4\pi\alpha(4-3\gamma)at^{1+b}} \quad (33)$$

3.2 Radiation Density:

$$\rho_r = \frac{b^2-3b-9}{at^2} - \frac{3m^2}{at^b} - \frac{2\exp t^{(-b+2)}}{-b+3} - \frac{b^2-3b-2bt}{3\pi\alpha(4-3\gamma)t^3} - \frac{2\exp t^{(-b+2)}(b-3t)}{(4-3\gamma)t^2(-b+3)} + \frac{3m^2}{2\pi\alpha(4-3\gamma)t^{1+b}} \quad (34)$$

3.3 Matter Density Parmeter:

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{9t^2}{b^2} \left[\frac{b^2-3b-2bt}{6\pi\alpha(4-3\gamma)t^3} + \frac{2\exp t^{(-b+2)}(b-3t)}{12\pi\alpha(4-3\gamma)t^2} - \frac{3m^2}{8\pi\alpha(4-3\gamma)t^{1+b}} \right] \quad (35)$$

$$\Omega_r = \frac{\rho_r}{3H^2} = \frac{9t^2}{b^2} \left[\frac{b^2-3b-9}{at^2} - \frac{3m^2}{at^b} - \frac{2\exp t^{(-b+2)}}{-b+3} - \frac{b^2-3b-2bt}{4\pi\alpha(4-3\gamma)t^3} - \frac{2\exp t^{(-b+2)}(b-3t)}{(4-3\gamma)t^2(-b+3)} + \frac{3m^2}{2\pi\alpha(4-3\gamma)t^{1+b}} \right] \quad (36)$$

3.4 Radition Density parameter

$$\Omega = \Omega_m + \Omega_r = \frac{18t^2}{b^2} \left\{ \frac{b^2-3b-2bt}{6\pi\alpha(4-3\gamma)t^3} + \frac{2\exp t^{(-b+2)}(b-3t)}{12\pi\alpha(4-3\gamma)t^2} - \frac{3m^2}{8\pi\alpha(4-3\gamma)t^{1+b}} + \frac{b^2-3b-9}{at^2} - \frac{3m^2}{at^b} - \frac{2\exp t^{(-b+2)}}{-b+3} - \frac{b^2-3b-2bt}{4\pi\alpha(4-3\gamma)t^3} - \frac{2\exp t^{(-b+2)}(b-3t)}{(4-3\gamma)t^2(-b+3)} + \frac{3m^2}{2\pi\alpha(4-3\gamma)t^{1+b}} \right\} \quad (37)$$

3.4 Case I: Dust Model

In order to investigate the physical behavior of the fluid parameters we consider the Particular case of dust ,when $\gamma = 0$. The hubble parameter, expansion scalar, deceleration Parmeter, anisotropic parameter, shear scalar are given by

$$H = \frac{b}{3t} \quad (38)$$

$$\theta = \frac{b}{t}$$

$$q = \frac{d}{dt} \frac{b}{3t} - 1 \quad (40)$$

$$q = -\frac{b}{3t^2} - 1 \quad (41)$$

$$\sigma^2 = \frac{1}{2} \frac{b^2}{a^2 t^2} \quad (42)$$

$$A_m = \frac{t^2}{a^2} \frac{X^2}{b^2 t^2} \quad (43)$$

The energy density and density parameters are

$$\rho_m = \frac{2b^2-3b-2bt}{24\pi a t^3} + \frac{4 \exp t^{-b+2}(b-3t)}{48\pi a t^2} - \frac{3m^2}{16\pi a t^{1+b}} \quad (44)$$

$$\rho_r = \frac{b^2-3b-9}{a t^2} - \frac{3m^2}{a t^b} - \frac{2 \exp t^{(-b+2)}(b-3t)}{-b+3} - \frac{b^2-3b-2bt}{12\pi a t^3} - \frac{2 \exp t^{(-b+2)}(b-3t)}{48\pi a t^2(-b+3)} + \frac{3m^2}{16\pi a t^{1+b}} \quad (45)$$

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{9t^2}{b^2} \left[\frac{b^2-3b-2bt}{24\pi a t^3} + \frac{2 \exp t^{(-b+2)}(b-3t)}{48\pi a t^2} - \frac{3m^2}{32\pi a t^{1+b}} \right] \quad (46)$$

$$\Omega_r = \frac{\rho_r}{3H^2} = \frac{9t^2}{b^2} \left[\frac{b^2-3b-9}{a t^2} - \frac{3m^2}{a t^b} - \frac{2 \exp t^{(-b+2)}(b-3t)}{-b+3} - \frac{b^2-3b-2bt}{4\pi a} - \frac{\exp t^{(-b+2)}(b-3t)}{2t^2(-b+3)} + \frac{3m^2}{8\pi a t^{1+b}} \right] \quad (47)$$

$$\Omega = \Omega_m + \Omega_r = \frac{18t^2}{b^2} \left\{ \frac{b^2-3b-2bt}{24\pi a t^3} + \frac{2 \exp t^{(-b+2)}(b-3t)}{48\pi a t^2} - \frac{3m^2}{32\pi a t^{1+b}} + \frac{b^2-3b-9}{a t^2} - \frac{3m^2}{a t^b} - \frac{2 \exp t^{(-b+2)}(b-3t)}{-b+3} - \frac{b^2-3b-2bt}{16\pi a} - \frac{\exp t^{(-b+2)}(b-3t)}{2t^2(-b+3)} + \frac{3m^2}{8\pi a t^{1+b}} \right\} \quad (48)$$

3.5 Case II: Zeldovich Universe

$\gamma = 1$ In this case, the Hubble parameter, Expansion scalar, deceleration parameter, anisotropic parameter, shear scalar are given by

$$H = \frac{b}{3t} \quad (49)$$

$$\theta = \frac{b}{t} \quad (50)$$

$$q = \frac{d}{dt} \frac{b}{3t} - 1 \quad (51)$$

$$q = -\frac{b}{3t^2} - 1 \quad (52)$$

$$\sigma^2 = \frac{1}{2} \frac{b^2}{a^2 t^2} \quad (53)$$

$$A_m = \frac{t^2}{a^2} \frac{X^2}{b^2 t^2} \quad (54)$$

$$\rho_m = \frac{2b^2-3b-2bt}{6\pi a t^3} + \frac{4 \exp t^{-b+2}(b-3t)}{12\pi a t^2} - \frac{3m^2}{4\pi a t^{1+b}} \quad (55)$$

$$\rho_r = \frac{b^2-3b-9}{a t^2} - \frac{3m^2}{a t^b} - \frac{2 \exp t^{(-b+2)}(b-3t)}{-b+3} - \frac{b^2-3b-2bt}{3\pi a t^3} - \frac{2 \exp t^{(-b+2)}(b-3t)}{t^2(-b+3)} + \frac{3m^2}{2\pi a t^{1+b}} \quad (56)$$

$$\Omega = \Omega_m + \Omega_r = \frac{18t^2}{b^2} \left\{ \frac{b^2-3b-2bt}{6\pi a t^3} + \frac{2 \exp t^{(-b+2)}(b-3t)}{12\pi a t^2} - \frac{3m^2}{8\pi a t^{1+b}} + \frac{b^2-3b-9}{a t^2} - \frac{3m^2}{a t^b} - \frac{2 \exp t^{(-b+2)}(b-3t)}{-b+3} - \frac{b^2-3b-2bt}{4\pi a} - \frac{2 \exp t^{(-b+2)}(b-3t)}{t^2(-b+3)} + \frac{3m^2}{2\pi a t^{1+b}} \right\} \quad (57)$$

Conclusion:

The constructed cosmological model is singularity free also we have discussed two different cases of the Universe. Case I for dust model $\gamma = 0$ we observe that in this case H is constant for $t \rightarrow \infty$ and deceleration parameter indicates $q = -\frac{b}{3t^2} - 1$ indicates that expansion of universe is accelerated. Averages mean parameter A_m is also non zero constant which show that model is anisotropic. Also the energy density for matter radiation vanishes for t tends to infinity and corresponding density parameters are Vanishes. In case II we obtain Zeldovich universe for $\gamma = 1$. For this universe also Hubble parameter and expansion scalar are positive value. The deceleration parameter we get negative value. That shows that universe is accelerating. The anisotropic mean parameter A_m show that the model is anisotropic.

The energy density and radiation density is positive value. $\frac{\sigma^2}{\theta^2} = \frac{\frac{1}{2} \frac{b^2}{a^2 t^2}}{\frac{b}{t}} = 0$ in all cases. Thus in all cases the model is anisotropic and accelerating which can be thought as of realistic model of the universe.

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