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Innovative Approaches to Partial Differential Equations Using the New Double Integral Transform (DIT)

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Abstract: This paper introduces a novel Double Integral Transform, referred to as the New DIT Transform. We provide the definition of the DIT Transform and its application to several special functions. Additionally, we demonstrate the use of the New DIT Transform method for solving Partial Differential Equations.

Keywords: Integral Transform, Partial Differential Equations.

Introduction: For solving partial differential equations (PDE)s, integral transforms are considered the most effective technique. Their significance stems from the fact that PDEs can be used to mathematically explain a variety of occurrences in mathematical physics and some other scientific fields [1,2,3,4,5,6,7,8,9,10,11,12,13]. These equations can also be transformed in order to find the exact solutions to PDEs using integral transforms. Because of the strength and simplicity of the transform techniques, scientists and researchers have worked very hard at studying and improving them.

Many integral transforms have been established and implemented to solve partial and integral differential equations; these transforms allow us to obtain the exact solutions of the target equations without needing linearization or discretization; they are applied to transform partial differential equations into ordinary ones, in the case of using single transformation, or into algebraic ones if we use double integral transformation. As examples, we mention Laplace transform [14], Fourier transform [15], Novel transform [16], M-transform [17], Sumudu transform [18], Natural transform [19], Elzaki transform [20], Kamal transform [21], the Aboodh transform [22], ARA transform [23] and ZZ transform [24].

The double transforms have also been widely applied to solving PDEs with unknown functions of two variables, and as a result, double transforms are considered very effective in handling PDEs compared to other numerical approaches [25,26,27,28]. In addition, extensions of the double transform have been developed in the relevant literature, such as the double Laplace transform, double Shehu transform [29], double Kamal transform [30], double Sumudu transform [31,32,33,34,35,36], double Elzaki transform [37], double Laplace–Sumudu transform [38] and ARA–Sumudu transform [39,40]. All these double transforms cited above can be considered special cases of the general double transform by Meddahi et al. [41], but sometimes we need to study special kinds of double transforms and compare them to consider the properties of each one, and decide on the best ones to use in handling new applications.

For all of these merits, we decided to establish new Double Integral Transform, so that we could reap the benefits of this transforms. We called this new approach the New DIT transform [42].

This paper is organised as follows. In second section we state definitions of some integral transforms in tabular form. In third section we have defined New Double Integral Transform called New DIT Transform. Formulae in table form of some elementary functions are summarised in fourth section.Applications of New DIT transform is presented in fifth section and conclusion has been given in sixth section.

II. DEFINITIONS OF SOME INTEGRAL TRANSFORMS

If f(t) be a function of exponential order α then we define the following transforms

Laplace transform	$L\{f\ (t\ )\} = \int_0^\infty f(t)e^{-ut}dt$
Sumudu transform	$S\ \{f\ (t\ )\} = \frac{1}{u}\int_0^\infty f(t)e^{-\frac{t}{u}}dt$
Elzaki Transform	$S\ \{f\ (t\ )\} = u\int_0^\infty f(t)e^{-\frac{t}{u}}dt$
Natural Transform	$N\ \{f\ (t\ )\} = R\ (s, u) = \frac{1}{u}\int_0^\infty f(t)e^{-\frac{st}{u}}dt$
Aboodh transform	$A\ \{f\ (t\ )\} = \frac{1}{u}\int_0^\infty f(t)e^{-ut}dt$
Mohand transform	$M\ \{f\ (t\ )\} = u^2\int_0^\infty f(t)e^{-ut}dt$
Mahgoub transforms	$M\ \{f\ (t\ )\} = u\int_0^\infty f(t)e^{-ut}dt$
Sawi transforms	$Sa\ \{f\ (t\ )\} = \frac{1}{u^2}\int_0^\infty f(t)e^{-\frac{t}{u}}dt$
Kamal transforms	$K\ \{f\ (t\ )\} = \int_0^\infty f(t)e^{-\frac{t}{u}}dt$
Pourreza transform	$HJ\ \{f\ (t\ )\} = u\int_0^\infty f(t)e^{-u^2t}dt$
G-_transform	$G\ \{f\ (t\ )\} = u^\alpha$

III A New DIT Transform

Inthissection,weintroduceanovelconceptofdoubletransformintwodimensionalspacescall edtheDIT Transform.

3.1 Definition: Let f (x, t) be a function of two variables x and t where $\mathbf{x, t \geq 0, p(s) \neq 0}$, $\mathbf{q(r) \neq 0}$and φ(s)and$\boldsymbol{\psi}$(r)arethetransformfunctionsforxand t,respectively. We define a NewDIT Transform of the function $\mathbf{f(x,t)}$ denoted by $\mathbf{F_D(s,r)}$ is defined by

$$\mathbf{F_D(s,r)} = \boldsymbol{T_2}\{\mathbf{f(x,t),(s,r)}\} = \mathbf{p(s)q(r)}\ \int_0^\infty\int_0^\infty \mathbf{e^{-\varphi(s)x}e^{-\psi(r)t}f(x,t)dxdt} \text{-------------(1)}$$

The inverse of the DIT Transform is given by

$$\mathbf{f(x,t)} = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}\frac{1}{p(s)}e^{\varphi(s)y}\varphi'(s)ds\,\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty}\frac{1}{q(r)}e^{\psi(r)y}\psi'(r)dr\ F_D(s,r)\boldsymbol{dr}.\text{------------(2)}$$

Where α and β are real constants.

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IV DIT Transform of Some Special Functions

Function $f(x,\mathrm{t})$	The New DIT Transform $\mathrm{T_2\{f(x,t)\}}$
1	$\dfrac{\mathrm{p(s)\ q(r)}}{\varphi(\mathrm{s})\psi(\mathrm{r})}$
$\mathrm{x.t}$	$\dfrac{\mathrm{p(s)q(r)}}{\varphi(\mathrm{s})^2\psi(\mathrm{r})^2}$
$\mathrm{x}^{\alpha}\mathrm{t}^{\beta}$	$\mathrm{p(s)q(r)}\dfrac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\varphi(\mathrm{s})^{\alpha+1}\psi(\mathrm{r})^{\beta+1}}$
$\mathrm{e}^{\mathrm{ax+bt}}$	$\dfrac{\mathrm{p(s)\ q(r)}}{(\varphi(\mathrm{s})-\mathrm{a})(\psi(\mathrm{r})-\mathrm{b})}$
$\mathrm{e}^{\mathrm{i(ax+bt)}}$	$\mathrm{p(s)q(r)}\dfrac{[\varphi(\mathrm{s})\psi(\mathrm{r})-\mathrm{ab}]+\mathrm{i}[\mathrm{b}\varphi(\mathrm{s})+\mathrm{a}\psi(\mathrm{r})]}{[\varphi(\mathrm{s})^2+(\mathrm{a})^2][\psi(\mathrm{r})^2+(\mathrm{b})^2]}$
$\{\cos(\mathrm{ax+bt})\}$	$\mathrm{p(s)q(r)}\dfrac{[\varphi(\mathrm{s})\psi(\mathrm{r})-\mathrm{ab}]}{[\varphi(\mathrm{s})^2+(\mathrm{a})^2][\psi(\mathrm{r})^2+(\mathrm{b})^2]}$
$\{\sin(\mathrm{ax+bt})\}$	$\mathrm{p(s)q(r)}\dfrac{[\mathrm{b}\ \varphi(\mathrm{s})+\mathrm{a}\ \psi(\mathrm{r})]}{[\varphi(\mathrm{s})^2+(\mathrm{a})^2][\psi(\mathrm{r})^2+(\mathrm{b})^2]}$
$\cosh(\mathrm{ax+bt})$	$\mathrm{p(s)q(r)}\dfrac{[\varphi(\mathrm{s})\psi(\mathrm{r})+\mathrm{ab}]}{[\varphi(\mathrm{s})^2-(\mathrm{a})^2][\psi(\mathrm{r})^2-(\mathrm{b})^2]}$
$\sinh(\mathrm{ax+bt})$	$\mathrm{p(s)q(r)}\dfrac{[\mathrm{b}\varphi(\mathrm{s})+\mathrm{a}\psi(\mathrm{r})]}{[\varphi(\mathrm{s})^2-(\mathrm{a})^2][\psi(\mathrm{r})^2-(\mathrm{b})^2]}$

V Applications

Theorem 1- Let f (x, t) be a function of two variables. If the first ordered partial derivative $\frac{\delta f}{\delta x}$and $\frac{\delta f}{\delta t}$ exists and$f$ (0, t) be given. $p(s), q(r), \varphi(s)$ *and* $\psi(r)$are positive real functions then

$$\tau_2\left\{\frac{\partial f}{\partial x}(x,t)\right\} = -p(s)\tau\{f(0,t)\} + \varphi(s)\tau_2\{f(x,t)\}$$

Where $\tau\{f(0,t)\}$ is the new general integral transform of the $f(0,t)$.

Proof: $\tau_2\left\{\frac{\partial f}{\partial x}(x,t)\right\} = \mathrm{p(s)q(r)}\int_0^\infty\int_0^\infty \frac{\delta f}{\delta x}\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}\mathrm{dxdt}$

$$= \mathrm{p(s)q(r)}\int_0^\infty\left(\int_0^\infty \frac{\delta f}{\delta x}\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{dx}\right)\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}dt$$

$$= \mathrm{p(s)q(r)}\int_0^\infty\left[\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\int_0^\infty\frac{\delta f}{\delta x}dx - \int_0^\infty\left(-\varphi(\mathrm{s})\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\int_0^\infty\frac{\delta f}{\delta x}dx\right)dx\right]\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}dt$$

$$= \mathrm{p(s)q(r)}\int_0^\infty[(-f(0,t)+\varphi(\mathrm{s}))\int_0^\infty \mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}f(x,t)dx]\,\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}dt$$

$$= -\mathrm{p(s)q(r)}\int_0^\infty f(0,t)\,\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}dt + \mathrm{p(s)q(r)}\varphi(\mathrm{s})\int_0^\infty\int_0^\infty \mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}f(x,t)dxdt$$

$$= -\mathrm{p(s)}\left[\mathrm{q(r)}\int_0^\infty f(0,t)\,\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}dt\right] + \varphi(\mathrm{s})\left[\mathrm{p(s)q(r)}\int_0^\infty\int_0^\infty \mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}f(x,t)dxdt\right]$$

$$= -\mathrm{p(s)}\tau\{f(0,t)\} + \varphi(s)\tau_2\{f(x,t)\}$$

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Theorem 2- Let $(x,$ t) be a function of two variables. If the first ordered partial derivative $\frac{\delta f}{\delta x}$and $\frac{\delta f}{\delta t}$ exists and$f$ (x, 0) be given. $p(s), q(r), \varphi(s)$ *and* $\psi(r)$ are positive real functions then

$$\tau_2\left\{\frac{\partial f}{\partial t}(x,t)\right\} = -\mathrm{q(r)}\tau\,\{f(x,0)\} + \psi(\mathrm{r})\tau_2\{f(x,t)\}$$

$\tau\,\{f(x,0)\}$ is the new general integral transform of the f (x, 0).

Proof::$\tau_2\left\{\frac{\partial f}{\partial t}(x,t)\right\} = \mathrm{p(s)q(r)}\int_0^\infty\int_0^\infty \frac{\delta f}{\delta t}\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}\mathrm{dxdt}$

$$= \mathrm{p(s)q(r)}\int_0^\infty\left(\int_0^\infty \frac{\delta f}{\delta t}\,\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}dt\;\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{dx}\right)\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}dx$$

$$= \mathrm{p(s)q(r)}\int_0^\infty\left[\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}\int_0^\infty\frac{\delta f}{\delta t}dt - \int_0^\infty\left(-\psi(\mathrm{r})\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}\quad\int_0^\infty\frac{\delta f}{\delta t}dt\right)dt\right]\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}dx$$

$$= \mathrm{p(s)q(r)}\int_0^\infty\left[(-f(x,0)) + \psi(\mathrm{r})\int_0^\infty \mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}f(x,t)dt\right]\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}dx$$

$$= -\mathrm{p(s)q(r)}\int_0^\infty f(x,0)\,\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}dx +$$
$$\mathrm{p(s)q(r)}\psi(\mathrm{r})\int_0^\infty\int_0^\infty \mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}f(x,t)dxdt$$

$$= -\mathrm{q(r)}\left[\mathrm{p(s)}\int_0^\infty f(x,0)\,\mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}dx\right] +$$
$$\psi(\mathrm{r})\left[\mathrm{p(s)q(r)}\int_0^\infty\int_0^\infty \mathrm{e}^{-\varphi(\mathrm{s})\mathrm{x}}\mathrm{e}^{-\psi(\mathrm{r})\mathrm{t}}f(x,t)dxdt\right]$$

$$= -\mathrm{q(r)}\tau\,\{f(x,0)\} + \psi(\mathrm{r})\tau_2\{f(x,t)\}$$

Example: Solve the Partial Differential equation $\frac{\delta f}{\delta x} = \frac{\delta f}{\delta t}$ with the initial conditions $f(0,t) = t\,, f(x,0) = x.$

Solution: Let$\frac{\delta f}{\delta x} = \frac{\delta f}{\delta t}$

Applying New DIT Transform, $\tau_2\left\{\frac{\partial f}{\partial x}(x,t)\right\} = \tau_2\left\{\frac{\partial f}{\partial t}(x,t)\right\}$

$$\Rightarrow -\mathrm{p(s)}\tau\,\{f(0,t)\} + \varphi(s)\tau_2\{f(x,t)\} = -\mathrm{q(r)}\tau\,\{f(x,0)\} + \psi(\mathrm{r})\tau_2\{f(x,t)\}$$

$$\Rightarrow\; -\mathrm{p(s)}\tau\,\{t\} + \varphi(s)\tau_2\{f(x,t)\} = -\mathrm{q(r)}\tau\,\{x\} + \psi(\mathrm{r})\tau_2\{f(x,t)\}$$

$$\Rightarrow\; -\mathrm{p(s)}\frac{q(r)}{\psi(\mathrm{r})^2} + \varphi(s)\tau_2\{f(x,t)\} = -\mathrm{q(r)}\frac{p(s)}{\varphi(s)^2} + \psi(\mathrm{r})\tau_2\{f(x,t)\}$$

$$\Rightarrow\;\; \tau_2\{f(x,t)\}[\varphi(s) - \psi(\mathrm{r})] = \frac{p(s)q(r)}{\psi(\mathrm{r})^2} - \frac{p(s)q(r)}{\varphi(s)^2}$$

$$\Rightarrow \tau_2\{f(x,t)\}[\varphi(s) - \psi(\mathrm{r})] = \; p(s)q(r)\left(\frac{1}{\psi(\mathrm{r})^2} - \frac{1}{\varphi(s)^2}\right)$$

$$\Rightarrow \tau_2\{f(x,t)\}[\varphi(s) - \psi(\mathrm{r})] = \; p(s)q(r)\left(\frac{\varphi(s)^2 - \psi(\mathrm{r})^2}{\varphi(s)^2\psi(\mathrm{r})^2}\right)$$

$$\Rightarrow \tau_2\{f(x,t)\} = \left(\frac{p(s)q(r)}{\varphi(s)^2\psi(\mathrm{r})^2}\right)[\varphi(s) + \psi(\mathrm{r})]$$

$$\Rightarrow \tau_2\{f(x,t)\} = \frac{p(s)q(r)}{\varphi(s)\psi(\mathrm{r})^2} + \frac{p(s)q(r)}{\varphi(s)^2\psi(\mathrm{r})}$$

$$\Rightarrow f(x,t) = x + t$$

VI Conclusion: In this paper we have defined the New DIT Transform and DIT transform of some useful functions in table form. We also applied New DIT Transform to some problems and obtained the results successfully.

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