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Bianchi Type III Cosmological Model With Electromagnetic Fields In $f(R,T)$ Theory of Gravitation

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Abstract: In this paper, we have studied the Bianchi type-III cosmological model interacting scalar and electromagnetic field in $f(R,T)$ theory of gravitation. We assume the electromagnetic vector potential in the form $V_i = [\alpha(x)V_1(t), V_2(t), V_3(t), V_4(t)]$, and by considering the general case. It is found that if the study is confined to the case of $f(R,T) = f_1(R) + \lambda f_2(R)$ .It is observed that the convergent and isotropic solution of the function can be evolved with vector potential in the constant form.

Keyword: Bianchi Type III, Electromagnetic Field, $f(R,T)$ theory of gravity, isotropy, constant vector potential.

1. Introduction: Many authors have investigated different problem within the scope of $f(R,T)$ theory. Tikekar and Patel [1] have obtained some exact Bianchi type-III cosmological solutions of massive string in the presence of magnetic field, Singh and Shri Ram [2] have presented a technique to generate new exact Bianchi type-II cosmological solutions of massive strings in the presence and absence of the magnetic field. Basically, two kinds of alternative reasons of accelerated expansion of the universe have been proposed for this unexpected observational phenomenon. One is Dark energy (DE) which has negative pressure and which induces a late-time accelerating cosmic expansion. The other is the modified gravity, which originate from the idea that the general relativity is incorrect in the cosmic scale and therefore need to be modified.

In order to explain the nature of the DE and accelerated expansion, a variety of theoretical models have been proposed in literature. The idea of introducing additional terms of the Ricci scalar to the Einstein-Hilbert action did not begin years ago with the $f(R)$ theory of gravity paper by Carroll *et.al* [6]. There are several modified gravity theories like $f(R)$ gravity formulated by Nojiri and Odintsov [7]. In $f(R,T)$ theory of gravity, cosmic acceleration may result not only due to geometrical contribution to the total cosmic energy density but it is also depends on matter contents. Pradhan [9] and Tripathy *et al.* [10] have been studied string cosmological models in the presence of the electromagnetic field. The modified theory $f(R,T)$ theory of gravitation is proposed by Harko T. *et al* [11,12] where R is the curvature scalar and T is the stress of energy momentum tensor. He explained the presence of a late time cosmic acceleration of the universe in $f(R)$ theory of gravity. Bijan Saha [15] has studied the interacting scalar and electromagnetic fields in Bianchi type I universe. Bianchi type-III cosmological model with a negative constant deceleration parameter in BD theory of gravity in

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presence of perfect fluid has been studied by Adhav *et al.*[13]. Katore et al. [14] explored plane symmetric space-time filled with dark energy models in BD theory. Solanke and Karade [16] have investigated Bianchi type-I and III universe field with perfect fluid and scalar field coupled with electromagnetic fields in theory of gravity. Mete and Mulestudied [17] Bianchi type-III charged fluid universe in Brans-Dicke theory of gravitation. Katore *et al.* [18] have investigated Bianchi type-I dark energy cosmological model with power-law relation in BD theory of gravitation. Chirde V.R *et. al* [19] studied Bianchi type-III and Kantowski Sachs cosmological model containing magnetic field with variable cosmological constant.Pawar, D.D, *et al.*,[20] explore Observational constraints on quark and strange quark matters in $f(R,T)$ theory of gravity. Our interest is to explore the role of scalar and electromagnetic field played in the amended $f(R,T)$ of gravity in other Bianchi types or other metric universe.In this paper, we discuss Bianchi type-III cosmological model with electromagnetic field in $f(R,T)$ theory of gravitation.

2.Gravitational Field Equations of $f(R,T)$ Gravity:

We consider Bianchi Type-III space time in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2dz^2, \tag{1}$$

where A , B and C are functions of t and m is a constant.

The field equations of $f(R,T)$ theories due to Harko [10] are deduced by varying the action

$$s = \int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{2}$$

Where L_m is lagrangian and the other symbols have their usual meaning.

Energy momentum tensor is given by $T_{ij} = L_m g_{ij} - 2\frac{\delta L_m}{\delta g^{ij}}$ (3)

Solanke and Karade[16] deduced G_j^μ and θ_{ij} and θ as follows

$$G_j^\mu = \frac{1}{f_R(R,T)}\left[g^{i\mu}\nabla_i\nabla_j f_R(R,T)\right] - \frac{1}{6f_R(R,T)}\left[f_R(R,T)R + f(R,T)\right]g_j^\mu$$
$$+\frac{\chi}{f_R(R,T)}\left[T_j^\mu - \frac{1}{3}Tg_j^\mu\right] + \frac{1}{3}\frac{f_T(R,T)}{f_R(R,T)}[T+\theta]g_j^\mu - \frac{f_T(R,T)}{f_R(R,T)}\left[T_j^\mu + \theta_j^\mu\right] \tag{4}$$

$$\theta_{ij} = -T_{ij} + 2\left[\frac{\delta L_m}{\delta g^{ij}} - g^{\alpha\beta}\frac{\delta^2 L_m}{\delta g^{ij}\delta g^{\alpha\beta}}\right] \tag{5}$$

$$\theta = I^2\ddot{\psi}\varphi_{,\eta}\varphi^{,\eta} \tag{6}$$

3.Electromagnetic field tensor F_{ij}:

we assume the electromagnetic vector potential in the form

$$V_i = [\alpha(x)V_1(t), V_2(t), V_3(t), V_4(t)], \tag{7}$$

Then it is easy to deduce

$$I = \left[\frac{\alpha^2 {V_1}^2}{A^2} + \frac{{V_2}^2}{B^2}e^{2mx} + \frac{{V_3}^2}{C^2} - {V_4}^2\right] \tag{8}$$

$$F_{14} = \alpha\dot{V}_1, F_{24} = \dot{V}_2, F_{34} = \dot{V}_3 \tag{9}$$

$$F_{ij}F^{ij} = -2\left[\frac{\alpha^2 {\dot{V}_1}^2}{A^2} + \frac{{\dot{V}_2}^2}{B^2}e^{2-x} + \frac{{\dot{V}_3}^2}{C^2}\right] \tag{10}$$

$$\varphi_i\varphi^i = -\dot{\varphi}^2 \tag{11}$$

The components of energy momentum tensors yields

$T_1^1 = \frac{1}{2}\frac{\alpha^2 \dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{2mx} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2} + \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{\alpha^2 V_1^2}{A^2}$ (12a)

$T_2^1 = \frac{\alpha\dot{V_1}\dot{v_2}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{\alpha V_1 V_2}{A^2}$ (12b)

$T_3^1 = \frac{\alpha\dot{V_1}\dot{v_3}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{\alpha V_1 V_3}{A^2}$ (12c)

$T_4^1 = \dot{\psi}\dot{\varphi}^2\frac{\alpha V_1 V_4}{A^2}$ (12d)

$T_2^2 = -\frac{1}{2}\frac{\alpha^2 \dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{2mx} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2} + \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}$ (12d)

$T_3^2 = \frac{\dot{V_2}\dot{v_3}}{B^2}e^{2mx} - \dot{\psi}\dot{\varphi}^2\frac{V_2 V_3}{B^2}e^{2mx}$ (12e)

$T_3^3 = -\frac{1}{2}\frac{\alpha^2 \dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{2mx} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2} + \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}$ (12f)

$T_4^4 = \frac{1}{2}\frac{\alpha^2 \dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{2mx} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2} - \frac{1}{2}\psi\dot{\varphi}^2 + \dot{\psi}\dot{\varphi}^2 V_4^2$ (12g)

$T = (\psi - I\dot{\psi})\dot{\varphi}^2$ (12h)

Similarly the components of tensors θ_j^i assumes the values

$\theta_1^1 = -T_1^1 - I\ddot{\psi}\dot{\varphi}^2\frac{\alpha^2 V_1^2}{A^2}$ (13a)

$\theta_2^1 = -T_2^1 - I\ddot{\psi}\dot{\varphi}^2\frac{\alpha V_1 V_2}{A^2}$ (13b)

$\theta_3^1 = -T_3^1 - I\ddot{\psi}\dot{\varphi}^2\frac{\alpha V_1 V_3}{A^2}$ (13c)

$\theta_4^1 = -T_4^1 - I\ddot{\psi}\dot{\varphi}^2\frac{\alpha V_1 V_4}{A^2}$ (13d)

$\theta_2^2 = -T_2^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{2mx}$ (13e)

$\theta_3^2 = -T_3^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2 V_3}{B^2}e^{2mx}$ (13f)

$\theta_3^3 = -T_3^3 - I\ddot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}$ (13g)

$\theta_4^4 = -T_4^4 + (\psi - I\dot{\psi})\dot{\varphi}^2 + I\ddot{\psi}\dot{\varphi}^2 V_4^2$ (13h)

$\theta = -I^2\ddot{\psi}\dot{\varphi}^2$ (13i)

The variation of the matter Lagrangian L_mBijan Saha [15]

$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(\sqrt{-g}F^{ij}\right) - (\varphi_{,j}\varphi^{,j})\dot{\psi}A^i = 0\,,\quad$ where$\dot{\psi} = \frac{\partial\psi}{\partial I}$

From (7) and (9), we get

for$i = 1, j = 4 \Rightarrow \left(\frac{\dot{V_1}}{V_1}\right)^{\cdot} + \frac{\dot{V_1}^2}{V_1^2} + \frac{\dot{V_1}}{V_1}\left[\frac{\dot{C}}{C} + \frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right] = \dot{\psi}\dot{\varphi}^2$ (14a)

for$i = 2, j = 4 \Rightarrow \left(\frac{\dot{V_2}}{V_2}\right)^{\cdot} + \frac{\dot{V_2}^2}{V_2^2} + \frac{\dot{V_2}}{V_2}\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right] = \dot{\psi}\dot{\varphi}^2$ (14b)

for$i = 3, j = 4 \Rightarrow \left(\frac{\dot{V_3}}{V_3}\right)^{\cdot} + \frac{\dot{V_3}^2}{V_3^2} + \frac{\dot{V_3}}{V_3}\left[\frac{\dot{B}}{B} + \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right] = \dot{\psi}\dot{\varphi}^2$ (14c)

for$i = 4, j = 1 \Rightarrow \alpha(x) = k_1 e^{mx}$where$k_1$ is constant. (14d)

for$i = 4, j = 4 \Rightarrow V_4 = 0\,,$ (14e)

4. Particular Case$f(R,T) = f_1(R) + \lambda f_2(T)$:

In this case we usefollowing notations

$f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = \dot{f}_1(R)$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = \lambda \dot{f}_2(T)$

Then (4) reduces to the form

$$G_j^\mu = \frac{1}{\dot{f}_1(R)}\left[g^{i\mu}\nabla_i\nabla_j \dot{f}_1(R)\right] - \frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R + f_1(R) + \lambda f_2(T)\right]g_j^\mu + \frac{\chi}{\dot{f}_1(R)}\left[T_j^\mu - \frac{1}{3}Tg_j^\mu\right] +$$
$$\frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]g_j^\mu - \frac{\lambda \dot{f}_2(T)}{\dot{f}_1(R)}\left[T_j^\mu + \theta_j^\mu\right] \quad (15)$$

Using (12) and (13), the field equations (15), yield

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \quad (16)$$
$$\frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \quad (17)$$
$$\frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \quad (18)$$

From (16-18), we can write

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \quad (19)$$

Above equation rewrite it as

$\frac{\dot{V}_1}{V_1} = \frac{\dot{V}_2}{V_2} = \frac{\dot{V}_3}{V_3} = \frac{\dot{h}_1}{h_1}$ say ,where h_1 is some unknown function of t. (20)

From (21) and (19), we get

$$\left(\frac{\dot{h}_1}{h_1}\right)^2 = \left(\frac{\dot{h}_1}{h_1}\right)^2 = \left(\frac{\dot{h}_1}{h_1}\right)^2 = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \quad (21)$$

Integrating (20) with respect to t, we get

$$V_1 = k_2h_1, \quad V_2 = k_3h_1, V_3 = k_4h_1 \quad (22)$$

wherek_2, k_3, k_4 are constants of integration.

Now our plan is to express the components of T_j^iin (12) in terms of T_4^4.

For this we consider the expression

$$\frac{\alpha^2{\dot{V}_1}^2}{A^2} + \frac{{\dot{V}_2}^2}{B^2}e^{2mx} + \frac{{\dot{V}_3}^2}{C^2} = \left[\frac{\alpha^2{V_1}^2}{A^2} + \frac{{V_2}^2}{B^2}e^{2mx} + \frac{{V_3}^2}{C^2}\right]\left(\frac{\dot{h}_1}{h_1}\right)^2 \text{ by} \quad (22)$$

$= I\left(\frac{\dot{h}_1}{h_1}\right)^2$by (7) and (14e)

$= I\dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I^2\ddot{\psi}\dot{\varphi}^2$ by (21) (23)

The components of T_j^i in (17) in terms of T_4^4 by using (20), (21) and (23)express as follows

$$T_4^4 = \frac{1}{2}I\dot{\psi}\dot{\varphi}^2 - \frac{1}{2}\frac{\lambda}{\chi}\dot{f}_2(T)I^2\ddot{\psi}\dot{\varphi}^2 - \frac{1}{2}\psi\dot{\varphi}^2 \quad (24a)$$
$$T_1^1 = -T_4^4 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{\alpha^2{V_1}^2}{A^2} \quad (24b)$$
$$T_2^1 = -\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{\alpha V_1 v_2}{A^2} \quad (24c)$$
$$T_3^1 = -\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{\alpha V_1 v_3}{A^2} \quad (24d)$$
$$T_4^1 = 0 \quad (24e)$$

$T_2^2 = -T_4^4 - \frac{\lambda}{\chi}\dot{f}_2(T)I\dot{\psi}\dot{\varphi}^2\frac{V_2{}^2}{B^2}e^{2m}$ (24f)

$T_3^2 = -\frac{\lambda}{\chi}\dot{f}_2(T)I\dot{\psi}\dot{\varphi}^2\frac{V_2 v_3}{B^2}$ (24g)

$T_3^3 = -T_4^4 - \frac{\lambda}{\chi}\dot{f}_2(T)I\dot{\psi}\dot{\varphi}^2\frac{V_3{}^2}{C^2}$ (24h)

$T = (\psi - I\dot{\psi})\dot{\varphi}^2$ (25)

We consider the non-vanishing components of Einstein tensors $G_1^1, G_2^2, G_3^3, G_4^1$ from (15)

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=\frac{\dot{A}}{A}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)]+\frac{\chi}{\dot{f}_1(R)}\left[T_1^1-\frac{1}{3}T\right]+$$
$$\frac{\lambda\dot{f}_2(T)}{3\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{3\dot{f}_1(R)}[T_1^1+\theta_1^1] \quad (26)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=\frac{\dot{B}}{B}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)]+\frac{\chi}{\dot{f}_1(R)}\left[T_2^2-\frac{1}{3}T\right]+$$
$$\frac{\lambda\dot{f}_2(T)}{3\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{3\dot{f}_1(R)}[T_2^2+\theta_2^2] \quad (27)$$

$$-\frac{m^2}{A^2}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{Ab}=\frac{\dot{C}}{C}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dD}-\frac{1}{6\dot{f}_1(R)}[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)]+\frac{\chi}{\dot{f}_1(R)}\left[T_3^3-\frac{1}{3}T\right]+$$
$$\frac{\lambda\dot{f}_2(T)}{3\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{3\dot{f}_1(R)}[T_3^3+\theta_3^3] \quad (28)$$

$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0$(29)

Integrating (29), we obtain

$A = k_5 B$wherek_5 is constant of integration(30)

Subtracting (26) from (25), (27) from (26) and (26) from (28) we get

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{C}}{C}\left[\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right]+\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=\frac{\chi}{\dot{f}_1(R)}[T_1^1-T_2^2]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_2^2+\theta_2^2)-(T_1^1+\theta_1^1)] \quad (31a)$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\left[\frac{\dot{}}{C}-\frac{\dot{B}}{B}\right]+\left(\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right)\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{m^2}{A^2}=\frac{\chi}{\dot{f}_1(R)}[T_2^2-T_3^3]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_3^3+\theta_3^3)-(T_2^2+\theta_2^2)] \quad (31b)$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{c}+\frac{\dot{B}}{B}\left[\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right]+\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{m^2}{A^2}=\frac{\chi}{\dot{f}_1(R)}[T_3^3-T_1^1]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_1^1+\theta_1^1)-(T_3^3+\theta_3^3)] \quad (31c)$$

Using (13) and (24) the equations (31) reduces

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{C}}{C}\left[\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right]+\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=0 \quad (32a)$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\left[\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right]+\left(\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right)\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{m^2}{A^2}=0 \quad (32b)$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{c}+\frac{\dot{B}}{B}\left[\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right]+\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{m^2}{A^2}=0 \quad (32c)$$

Let us choose power law form of metric potential,Katore *et al.* [19] given by

$B = at^n$and$C = \beta t^n$ (33)

The power law relation between $f_1(R)$and the scale factor$a$is $f_1(R) = \eta a^m$, where a is constant of proportionality. The average scalar factor is given by

$a = (ABC)^{\frac{1}{3}} = k_5 t^n,$ (34)

where$k_5 = \alpha^{\frac{2}{3}}\beta^{\frac{2}{3}}$,is a constant.

Hence scalar field $f_1(R)$ is obtained as

$f_1(R) = k_5 t^{mn} \Rightarrow f_1(R) = k_5 t^s$, where $s = mn$ is a constant.

From (26) we have

$$\dot{\psi}\dot{\varphi}^2 = \left(\frac{\dot{h}_1}{h_1}\right)^2 + \frac{\lambda}{\chi}\dot{f}_2(T) I \ddot{\psi}\dot{\varphi}^2 \quad (35)$$

Inserting (35) in (34) we have

$$\left(\frac{\dot{h}_1}{h_1}\right)^{\cdot} + \frac{\dot{h}_1}{h_1}\left[\frac{\dot{A}}{A}\right] = \frac{\lambda}{\chi}\dot{f}_2(T) I \ddot{\psi}\dot{\varphi}^2 \quad (36)$$

If we confine the function ψ as linear function $\ddot{\psi} = 0$ or $\psi = k_{22} I + k_{23}$ then the equation (36) has perfect solution

$$h_1 = k_{25} exp\left\{k_{24}\int \frac{1}{A} dt\right\} \quad (37)$$

The metric in (1), with the help of (33), can be redefined in the form

$$ds^2 = \alpha^2 t^{2n}(dx^2 + e^{-2mx})dy^2 + \beta^2 t^{2n} dz^2 - dt^2. \quad (38)$$

The physical quantities of observational interest in cosmology are,

The spatial volume is obtained as$V = \sqrt{-g} = (\alpha^2 \beta t^{2n}) e^{-mx}$. (39)

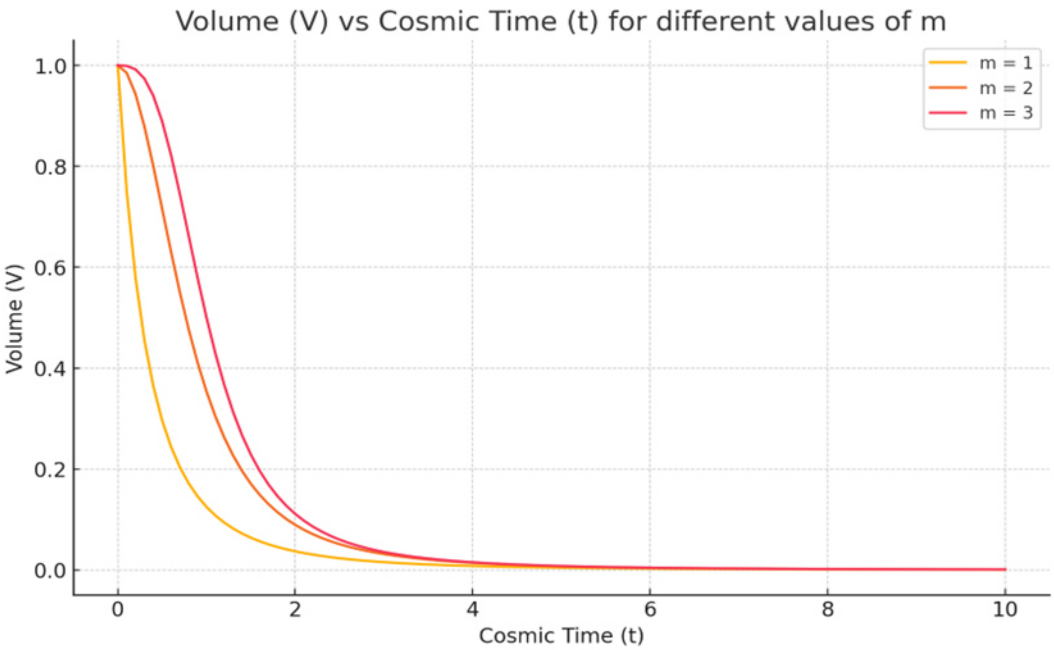
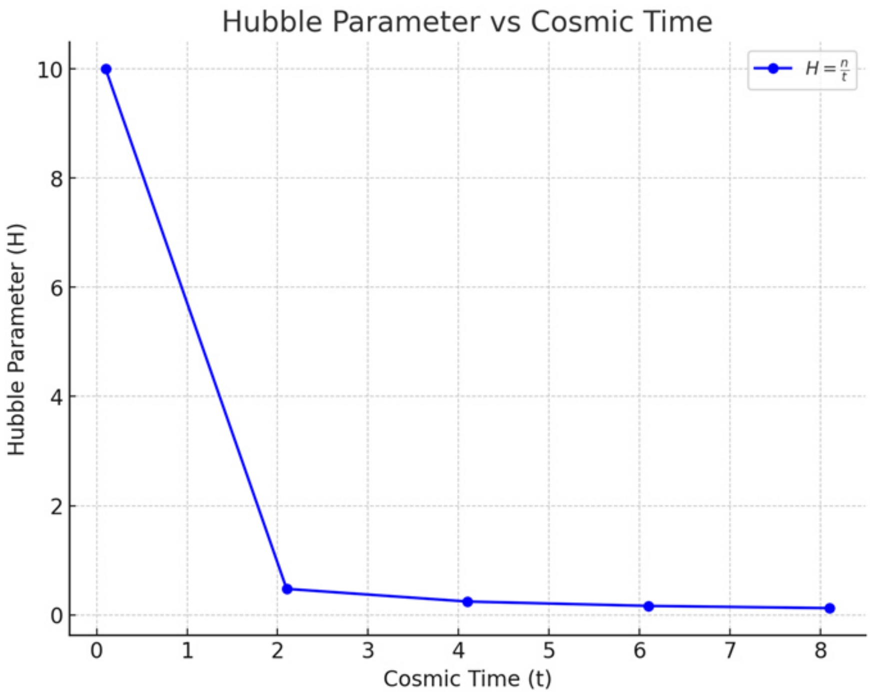


Figure 4.1: Volume versus cosmic time

The volume starts from a high value at t=0 and decreases over time in this form (due to the negative exponent). Physically, this represents a scenario of a contracting universe, where the cosmic volume shrinks over time or it could reflect expansion from small scales to large scales.

The expansion scalar become$\theta = 3H = \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{3n}{t}$, (40)

The shear scalar is given by$\sigma^2 = \frac{1}{2}\sum_{i=1}^{3} {H_i}^2 - \frac{\theta^2}{\sigma} = 0$ (41)



The mean anisotropic parameter A_m as $\Delta = \frac{1}{2}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 0$ (42)

The mean Hubble parameter $H = \frac{n}{t}$. (43)

Figure 4.2: Hubble parameter versus cosmic time. The graph shows that the Hubble parameter H decreases with the increase in cosmic time t. At early times (close to the Big Bang), the Hubble parameter is very large, indicating rapid expansion of the universe. As time progresses, the expansion rate slows down, leading to a decrease in H. The plot follows an inverse relationship, meaning the expansion rate is inversely proportional to cosmic time. This is characteristic of a matter-dominated universe or certain cosmological models where expansion slows over time.

The deceleration parameter as $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{1-n}{n}$ (44)

It is observed that the deceleration parameter q comes out to be constant and it depends on the value of n. For $n > 1$ the sign of q become negative which correspond to the standard accelerating behavior of the universe. At $n = 1, q$ becomes zero, indicating constant expansion rate neither acceleration nor deceleration. For is $n > 1, q$ is positive, showing deceleration of the universe's expansion.

The cosmic Jerk parameter is given as, $= q + 2q^2 - \left(\frac{\dot{q}}{H}\right) = \frac{(n-1)(n-2)}{n^2}$. (45)

At early cosmic times (small n): The jerk parameter j can be large and either positive or negative, depending on n. It indicates rapid changes in acceleration, signifying dynamic transitions in the early universe.

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For $n = 1, : j = 0$, which implies a scenario where the rate of acceleration does not change a constant acceleration or deceleration phase.For large n (at late times): As n→∞, the jerk parameter tends toward 1, consistent with the behavior of a cosmological constant-dominated, smooth accelerating universe.

5. Conclusion:

In this paper, we have presented Bianchi Type-III charged fluid universe in $f(R,T)$theory of gravitation in presence electromagnetic field. The volume starts from a high value at $t = 0$ and decreases over time in this form (due to the negative exponent). Physically, this represents a scenario of a contracting universe, where the cosmic volume shrinks over time or it could reflect expansion from small scales to large scales. It is observed that Hubble's parameter H vanishes with extremely large value and continue to decrease with time. At early times (close to the Big Bang), the Hubble parameter is very large, indicating rapid expansion of the universe. As time progresses, the expansion rate slows down, leading to a decrease in H. The scalar expansion and the physical parameters energy density and pressure diverge at $t = 0$ and they all vanish as t approaches to infinity. The scalar field increases with time and at $t = 0$, it vanishes. It is observed that the deceleration parameter q comes out to be constant and it depends on the value of n. For $n > 1$, the sign of q become negative which correspond to the standard accelerating behavior of the universe. At $n = 1$, q becomes zero, indicating constant expansion rate neither acceleration nor deceleration. For is $n > 1$, q is positive, showing deceleration of the universe's expansion. The recent observations of type Ia supernovae reveal that the present universe is accelerating and the value of deceleration parameter lies somewhere in the range $-1 < q < 0$. It follows that one can choose the value of n in the range $0 < n < 1$ to ensure that the derived model is consistent with observations. At Early times, we get large and variable jerk, showing rapid changes in acceleration. At $n = 0$, it indicates constant acceleration or deceleration. At late times, it reflect a stable, smooth acceleration like in the Lambda-CDM model.

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