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Plane symmetry Universe with Holographic Dark energy in Lyra Geometry

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Abstract: This paper deals with the plane symmetric cosmological model filled with interacting Dark matter and Holographic dark energy in Lyra geometry. To get the solutions of the field equations, we consider the scalar factor of the form $a = \frac{-1}{t} + t^2$, $t>1$. The physical and geometrical aspects of the model are studied in detail.

Keywords: Plane symmetric space-time, Interacting dark fluids, deceleration parameter, State finder parameters, Coincidence problem, Lyra Geometry.

1 Introduction:

Therecent cosmological observations of Type Ia supernovae (SNeIa) [1-2] indicate that the universe is currently accelerating. These results, when combined with the observations of cosmic microwave background (CMB) []3-4] and large scale structure (LSS) [5-6] strongly suggest that the universe is spatially flat and dominated by an exotic component with large neg -ative pressure called as dark energy (DE) [7-9]. Wilkinson Microwave Anisotropy Probe (WMAP) shows that dark energy occupies about 73 % of the energy of our universe and dark matter occupies about 23 % whereas the baryon matter occupies only about 4 % of the total energy of the universe.

There are many candidates for dark energy namely the quintessence scalar field models [10], the phantom field [11], K-essence [12], tachyon field [13], quintom [14], the dark energy models including Chaplygin gas [15] and so on. Recently, Bamba et al. [16] have reviewed different dark energy cosmologies (isotropic) with early deceleration and late time acceleration. They have studied f(R)gravity, f(R,T)gravity, f(T)gravity, Scalar field theory, Holographic dark energy, coupled dark energy and ΛCDM cosmological models representing the acceler- ating expansion with the quintessence/phantom nature in details along with Cosmography tests. Among the many different approaches to describe the dark cosmological sector, so called holographic dark energy models have received considerable attention[17-18]. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and related to the area of its boundary [19]. Based on this principle, a field theoretical relation between a short distance (ultraviolet) cutoff and a long distance (infrared) cutoff was established [17]. This relation ensures that the energy in a box of size L does not exceed the energy of a black hole of the same size. Applied to the dynamics of the Universe, L has to be a cosmological length scale. Different choices of this cutoff scale result in different dark energy models. If one identifies L with the Hubble radius H^{-1}, the resulting dark energy density, corre sponding to the ultra-violet cutoff, will be close to the ob served effective cosmological constant. Li [18] has subsequently shown the possibilities of the particle and the event horizons as the IR cutoff length and he found that apparently only a future event horizon cutoff can give a viable dark energy model. However, afterwards it was recognized, that a cutoff at the Hubble scale may well result in a realistic cosmological dynamics provided only, that an interaction in the dark sector is admitted [20]. Holographic dark energy models have been tested and constrained by various astronomical observations [21]. Aspecial class are models in which holographic DE is allowed to interact with DM [22-26]. Recently, Sarkar [27-29] have studied non-interacting holographic dark energy with linearly varying deceleration parameter in Bianchi type-I and V universe and interacting holographic dark energy in Bianchi type-II respectively.

The restriction of general relativity in giving satisfactory explanation of evolution of phase of the universe has been led cosmologists to acquire various hypothesis as well as study their inference in this context. These hypothesis contain those allocating i) the time-dependence of the cosmological term and the gravitational constant ii) other physical or geometrical field with the universe iii) modified theories of gravity. Such theories are expected to disclose a number of aspects of physical and mathematical interests associated with them. To generalize the objective of geometrizing the gravitation, to involve an electromagnetism's geometrical description, many researchers have used their effects as the Einstein's theory of GR has based on the geometrical description of gravitation. Weyl [30] had suggested a new kind of gauge theory which includes a metric tensor to geometrize the gravitation and the electromagnetism. Lyra [31] putforword a geometry, called as Lyra geometry, which is nothing but the modification of Riemannian geometry in which gauge function having structureless manifold is presented, as a result of which the cosmological constant appears naturally from the geometry. Sen and Dunn [32] and Sen [33] developed a new scalar tensor-theory based on Lyra geometry and similar to Einstein theory of GR. Using constant displacement vector, cosmological model based on Lyra geometry was studied by Soleng [34]. Gad [35] presented an axially symmetric cosmological mesonic stiff fluid model in Lyra's geometry. Adhav [36] studied LRS Bianchi type–I universe with anisotropic dark energy cosmological model with early deceleration as well as late-time acceleration in Lyra geometry. Pradhan and Singh [37] studied anisotropic Bianchi type-I string cosmological model in normal gauge for Lyra geometry with constant deceleration parameter. Shri Ram [38] investigated a Kantowski-Sachs universe with anisotropic dark energy in Lyra geometry.

From above discussion, in this paper we studied, the plane symmetric cosmological model filled with interacting Dark matter and Holographic dark energy in Lyra geometry. To get the solutions of the field equations, we consider the scalar factor of the form $a=\frac{-1}{t}+t^2$, $t>1$. The physical and geometrical aspects of the model are studied in detail.

2. Metric and Field Equations:

We consider the plane symmetric metric in the form as

$$ds^2 = dt^2 - A^2(dx^2+dy^2) - B^2dz^2 \quad , \tag{1}$$

where A and B are the scale factors and functions of the cosmic time t only .

The field equation in normal gauge for Lyra geometry is given by Sen [47] as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\left(\phi_i\phi_j - \frac{1}{2}g_{ij}\phi_\alpha\phi^\alpha\right) = {}^{m}T_{ij} + {}^{de}T_{ij} \, , \tag{2}$$

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where R_{ij} is the Ricci tensor, R is the Ricci scalar, ϕ_i refers to the time-varying displacement field vector, which is defined as,

$$\phi_i = (0,0,0,\beta), \tag{3}$$

where β is called the displacement vector.

The energy-momentum tensor for cold dark matter and Holographic dark energy are given by

$$^{m}T_{ij} = \rho_m u_i u_j \tag{4}$$

and $$^{\Lambda}T_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j - g_{ij} p_\Lambda , \tag{5}$$

together with $g_{ij}u^i u^j = 1$, where u^i is the flow vector of the fluid, ρ_m is the energy density of dark matter and ρ_Λ, p_Λ are the energy density and pressure of holographic dark energy respectively.

Using the co-moving co-ordinate system, the non-vanishing components of $^{\Lambda}T_i^j$ can be obtained as

$$^{\Lambda}T_0^0 = \rho_\Lambda, \quad ^{\Lambda}T_1^1 = {}^{\Lambda}T_2^2 = {}^{\Lambda}T_3^3 = -p_\Lambda, \quad ^{\Lambda}T_i^j = 0 \text{ for } i \neq j . \tag{6}$$

The field equations (2) for metric (1), with the help of equations (3) for (6) can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -p_\Lambda , \tag{7}$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{3}{4}\beta^2 = -p_\Lambda , \tag{8}$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{3}{4}\beta^2 = \rho_\Lambda + \rho_m , \tag{9}$$

$$\frac{3}{2}\beta\dot{\beta} + \frac{3\beta^2}{2}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0 . \tag{10}$$

where overhead dot $(\cdot)$ represents derivative with respect to time t.

Further we assume that the interaction between two perfect fluid, dark matter (pressureless) and holographic dark energy. We consider exchange of energy between these components in such matter that continuity equations for holographic dark energy and dark matter are given by

$$\dot{\rho}_m + \left(\frac{\dot{V}}{V}\right)\rho_m = Q , \tag{11}$$

$$\dot{\rho}_\Lambda + \left(\frac{\dot{V}}{V}\right)(1 + w_\Lambda)\rho_\Lambda = -Q . \tag{12}$$

The equation of state parameter (EoS) for holographic dark energy is given by $w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$. The quantity $Q > 0$ denotes the interaction between the dark energy components. Wetterich [10]introduced models featuring an interacting matter-dark energy and first used by Horvat [39] alongside the holographic dark energy. Although this expression for interacting term may looks purely phenomenological. Continuity equation (11) and (12) imply that the interacting term (Q) should be proportional to a quantity with unit of inverse time i.e. $Q \propto 1/t$. This occurred due to choosing H and Q term and is motivated purely by

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mathematical simplicity. To form energy density, any combination of dark matter and dark energy can be expressed phenomenological in form such as [40-41].

$$Q = 3b^2 H\rho_m = b^2 \frac{\dot{V}}{V}\rho_m , \qquad (13)$$

where b^2 is coupling constant. To avoid the coincidence problem, Cai and Wang [42] considered same relation for interacting phantom dark energy and dark matter.

$$\rho_m = \rho_0 V^{(b^2-1)} , \qquad (14)$$

where $\rho_0 > 0$ is a real constant of integration.

Using equations (13) and (14), we get the interacting term as

$$Q = 3\,\rho_0 b^2 H V^{(b^2-1)}. \qquad (15)$$

3. Cosmological Solutions:

Here, we have given four equations (7)-(10) in six variables A, B, p_Λ, ρ_Λ, ρ_m, β. Hence to obtain the solutions from these system of equations, one extra condition is required. Therefore, we consider scale factor of the form

$$a = -1/t + t^2 , \qquad (16)$$

which leads to the variable deceleration parameter.

Recently, scale factor of the form (16) is used by Pradhan [43], to discuss dark energy model with anisotropic fluid in Bianchi type-VI_0 space-time. The relation (16) is also used by Pradhan et. al. [44], to study Bianchi type-I cosmological model in scalar tensor theory of gravitation. The aim of choosing such scalar factor is that the universe is accelerating phase at present as well as decelerating expansion in past. Also, the transition redshift from decelerating expansion to accelerated expansion is about 0.5. So, in general, the decelerating parameter is not a constant but variable. By above choice of scalar factor yields a time dependent DP

Subtracting equation (7) from equation (8), we obtain

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} = 0 . \qquad (17)$$

After simplification, equation (17) yields to

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0 . \qquad (18)$$

After solving equation (18), we can write the metric functions A and B explicitly as

$$A = c_2^{1/3}\left(\frac{-1}{t} + t^2\right)\exp\left(\frac{c_1}{18}\left(\left[-\log(t^2+t+1)\right] + 2\log(1-t) + 2\sqrt{3}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right)\right), \qquad (19)$$

$$B = c_2^{-2/3}\left(\frac{-1}{t} + t^2\right)\exp\left(\frac{-2c_1}{18}\left(\left[-\log(t^2+t+1)\right] + 2\log(1-t) + 2\sqrt{3}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right)\right), \qquad (20)$$

where $c_1 > 0$ and c_2 are real constants of integration.

The solution of equation (10) using equations (19), (20) is given by

$$\beta = \frac{c}{(-1/t + t^2)^3}, \tag{21}$$

The volume is defined and obtained as

$$V = a^3 = \left(-\frac{1}{t} + t^2\right)^3. \tag{22}$$

The mean Hubble parameter is defined and obtained as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{2t^3+1}{t(t^3-1)}. \tag{23}$$

Using equation (22) in equations (14) and (15), we get the energy density of dark matter and the interacting term as

$$\rho_m = \rho_0 (-1/t + t^2)^{3(b^2-1)}, \tag{24}$$

$$Q = b^2 \rho_0 \frac{(2t^3+1)}{t(t^3-1)} (-1/t + t^2)^{3(b^2-1)}. \tag{25}$$

Using equations (19), (20) and (24) in the equation (9), we obtain the energy density of holographic dark energy as

$$\rho_\Lambda = \frac{3(2t^3+1)^2}{t^2(t^3-1)^2} - 3\left(\frac{c_1}{18}\left(\left[\frac{-2t-1}{t^2+t+1}\right] + \frac{2}{t-1} + \frac{3}{t^2+2t+1}\right)\right)^2 - \rho_0(-1/t+t^2)^{3(b^2-1)} - \frac{3c^2}{4(-1/t+t^2)^6}. \tag{26}$$

Using equations (19), (20) in the linear combination of equations (7)-(8), we obtain the pressure of holographic dark energy as

$$p_\Lambda = -\frac{4}{t^2} - \frac{(2t^3+1)^2}{t^2(t^3-1)^2} - 3\left(\frac{c_1}{18}\left(\left[\frac{-2t-1}{t^2+t+1}\right] + \frac{2}{t-1} + \frac{3}{t^2+2t+1}\right)\right)^2 - \frac{c_1}{18}\left(\left[\frac{-2t-1}{t^2+t+1}\right] + \frac{2}{t-1} + \frac{3}{t^2+2t+1}\right)\frac{(2t^3+1)}{t(t^3-1)} - \frac{c_1}{9}\left(\frac{2t^2+2t+1}{(t^2+t+1)^2} - \frac{2}{(t-1)^2} - \frac{6(t-1)}{(t^2+2t+1)}\right) - \frac{3c^2}{4(-1/t+t^2)^6}. \tag{27}$$

The EoS parameter of holographic dark energy is given by

$$w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}.$$

$$w_\Lambda = \frac{-\frac{4}{t^2} - \frac{(2t^3+1)^2}{t^2(t^3-1)^2} - 3\left(\frac{c_1}{18}\left(\left[\frac{-2t-1}{t^2+t+1}\right] + \frac{2}{t-1} + \frac{3}{t^2+2t+1}\right)\right)^2 - \frac{c_1}{18}\left(\left[\frac{-2t-1}{t^2+t+1}\right] + \frac{2}{t-1} + \frac{3}{t^2+2t+1}\right)\frac{(2t^3+1)}{t(t^3-1)} - \frac{c_1}{9}\left(\frac{2t^2+2t+1}{(t^2+t+1)^2} - \frac{2}{(t-1)^2} - \frac{6(t-1)}{(t^2+2t+1)}\right) - \frac{3c^2}{4(-1/t+t^2)^6}}{\frac{3(2t^3+1)^2}{t^2(t^3-1)^2} - 3\left(\frac{c_1}{18}\left(\left[\frac{-2t-1}{t^2+t+1}\right] + \frac{2}{t-1} + \frac{3}{t^2+2t+1}\right)\right)^2 - \rho_0(-1/t+t^2)^{3(b^2-1)} - \frac{3c^2}{4(-1/t+t^2)^6}}. \tag{28}$$

The coincidence parameter is the ratio of two energies density i.e. the ratio of dark matter energy density to the dark energy density is given by

$$\bar{r} = \frac{\rho_m}{\rho_\Lambda}$$

$$\bar{r} = \frac{\rho_0(-1/t+t^2)^{3(b^2-1)}}{\frac{3(2t^3+1)^2}{t^2(t^3-1)^2} - 3\left(\frac{c_1}{18}\left(\left[\frac{-2t-1}{t^2+t+1}\right]+\frac{2}{t-1}+\frac{3}{t^2+2t+1}\right)\right)^2 - \rho_0(-1/t+t^2)^{3(b^2-1)} - \frac{3c^2}{4(-1/t+t^2)^6}}.$$

(29)

The anisotropy parameter is defined and given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2X^2t^8}{3(t^3-1)^4(1+2t^3)^2}. \quad (30)$$

The deceleration parameter (q)is defined and given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{2(t^3-1)^2}{(2t^3+1)^2}. \quad (31)$$

The expansion Scalar (θ) is defined and given by

$$\theta = 3H = \frac{3(1+2t^3)}{t(t^3-1)}. \quad (32)$$

4 Statefinder Diagnostic

For the purpose of describing the current cosmic acceleration, several models of dark energy have been suggested by many authors. It is important to differentiate these various dark energy models. So, a new pair called as statefinder diagnostic pair $\{r,s\}$has been introduced by Sahni et al. [45]. The parameters r and s depend on a (scale factor) and are dimensionless, so $\{r,s\}$ is a geometric diagnostic. The Statefinder parameter $\{r,s\}$ is defined as,

$$r = \frac{\dddot{a}}{aH^3} \text{ and } s = \frac{r-1}{3(q-1/2)}. \quad (33)$$

Sahni and his co-workers are in large to allow curved universe models with the help of statefinder parameter, can be distinguish between various form of DE. For example phantom, chaplygin gas and quintessence models approach to ΛCDM $\{r,s\}_{\Lambda CDM} = \{1,0\}$. Trajectories of chaplygin gas models lie in the region$s<0, r>1$whereas in the region $s>0, r<1$,trajectories of phantom and quintessence model lie.

The *s-r* plane containing trajectories which corresponds to other cosmological models show qualitatively different behaviors. A fixed point $\{r,s\}|_{\Lambda CDM} = \{1,0\}$ shown in the fig. 6 corresponds to flat ΛCDM model. A good way which is provided for demonstration of 'distance' of this model from ΛCDM, depart a given dark energy model from the fixed point [46]. For differentiation amongst a large variety of the DE models including quintessence, cosmological constant, the Chaplygin gas, interacting dark energy models and braneworld models, the statefinder may prove to be successful tool [47].

Here we obtain the statefinder parameter as

$$r = 55 - 90\frac{e^{6t}-1}{e^{6t}} + 36\left(\frac{e^{6t}-1}{e^{6t}}\right)^2, \quad s = \frac{54 - 90\frac{e^{6t}-1}{e^{6t}} + 36\left(\frac{e^{6t}-1}{e^{6t}}\right)^2}{-\frac{9}{2}+\frac{18}{e^{6t}}}. \tag{34}$$

5. Discussion

1. The deceleration parameter (q): - The sign of q denotes whether the model inflates or not. A positive sign of q indicates the decelerating model whereas the negative sign of q denotes accelerating model. Recent observational data suggest that the universe is in phase of accelerating expansion. The evolution of the decelerating parameter is as shown in fig. 1. It clearly indicates that our model is in accelerating phase. As $t \to 0$, $q \to -2$ and after some time $q \to 0.5$ and remain as it is for long time

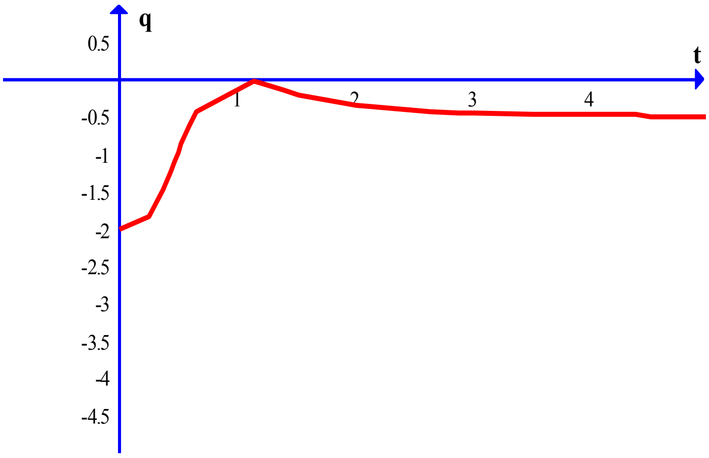

Fig. 1:- The deceleration parameter (q) v/s time (t)

2. The anisotropy parameter of the expansion (Δ): - The dynamical behavior of the anisotropic parameter is as shown in fig. 2. It is observed that the anisotropy parameter $\Delta \to 0$ as $t \to 0$ i.e. the model is isotropy for initial time. After some time, $\Delta \to \infty$ i.e. model convert to anisotropy.

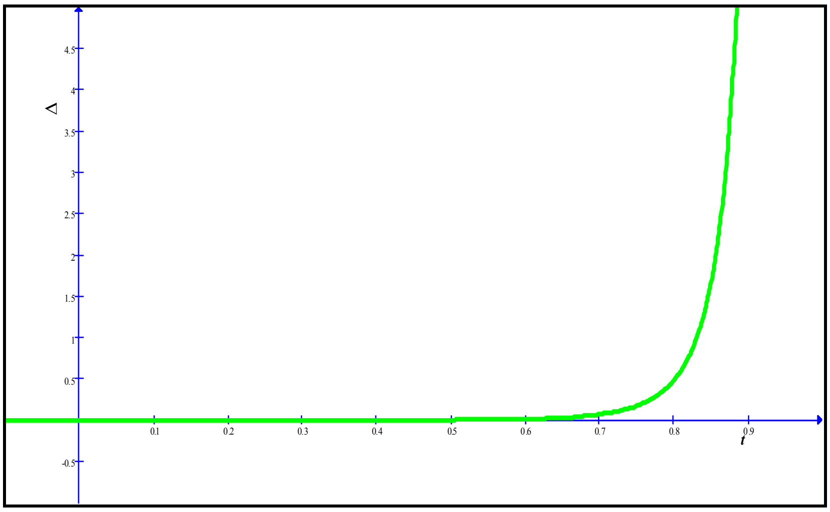

Fig. 2:- The anisotropic parameter (Δ) v/s time (t)

3. The EoS parameter (w_Λ): - Fig. 3 indicates that $t \to 0$, $w_\Lambda \to \infty$ .As time increases w_Λ

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starts from phantom region. After some finite time, it approaches to $w_\Lambda = -1$ (i.e. cosmological constant (Λ)), which denotes that our model reduces to ΛCDM. Then, it enters into quintessence region ($-1 < w_\Lambda < -1/3$) for further time t, whichmatches with the present-day cosmological observation. In fig. 3, red line denotes thegraph of w_Λ.

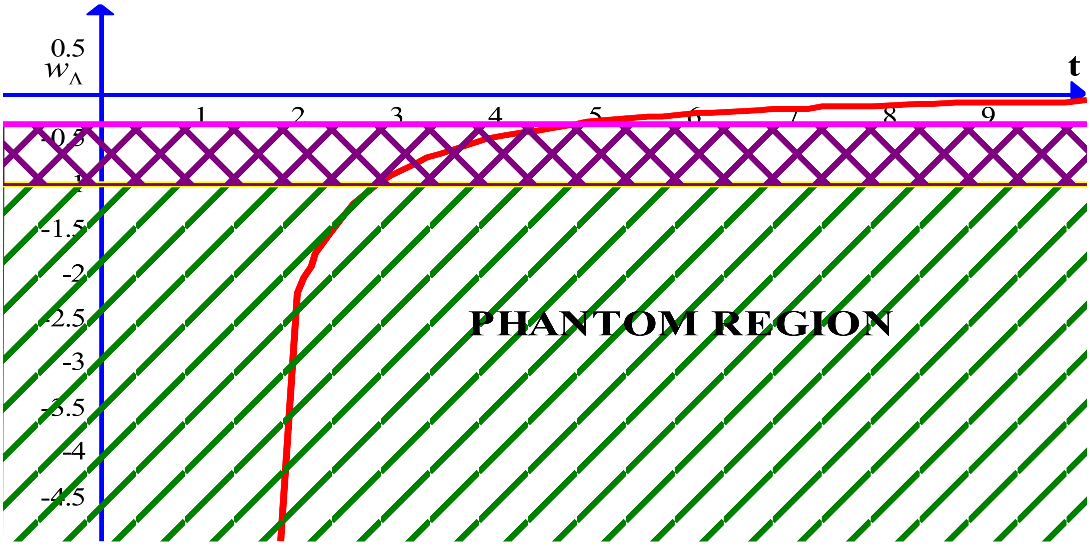

Fig: 3- The EoS parameter (w_Λ) v/s time (t)

4. The displacement vector (β):-The evolution of displacement vector (β) is as shown in fig. 4. The present value of the displacement vector is positive. It is observe that as $t \to 0$, $\beta \to \infty$ and as $t \to \infty$, $\beta \to 0$ for late time. So, Lyra geometry is physically meaningful for finite time, but for very large time it is converted to general relativity (GR).

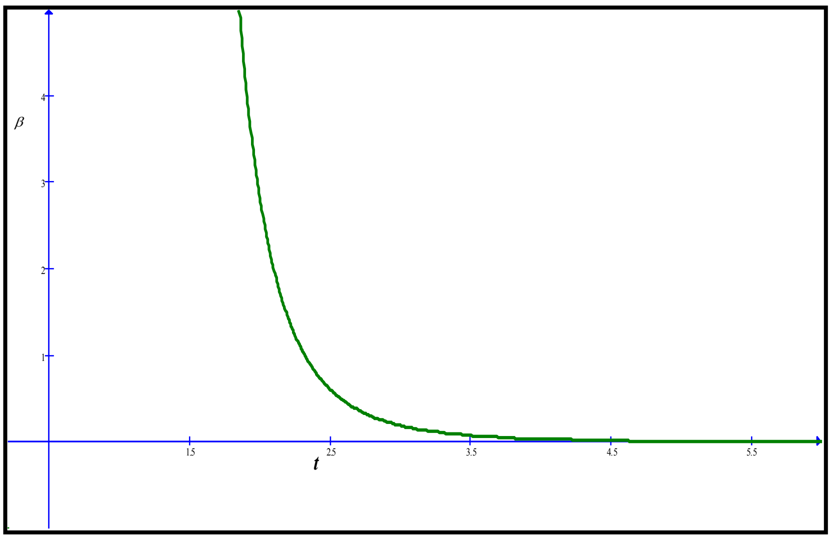

Fig. 4:- The displacement vector (β) v/s time (t)

5. The statefinder parameter:-Figure 5 shows the evolving trajectory of this scenario in the s-r plane which is quite different from those of other dark energy models. We hope that the future high precision observations will be capable of determining these statefinder parameters and consequently explore the nature of dark energy

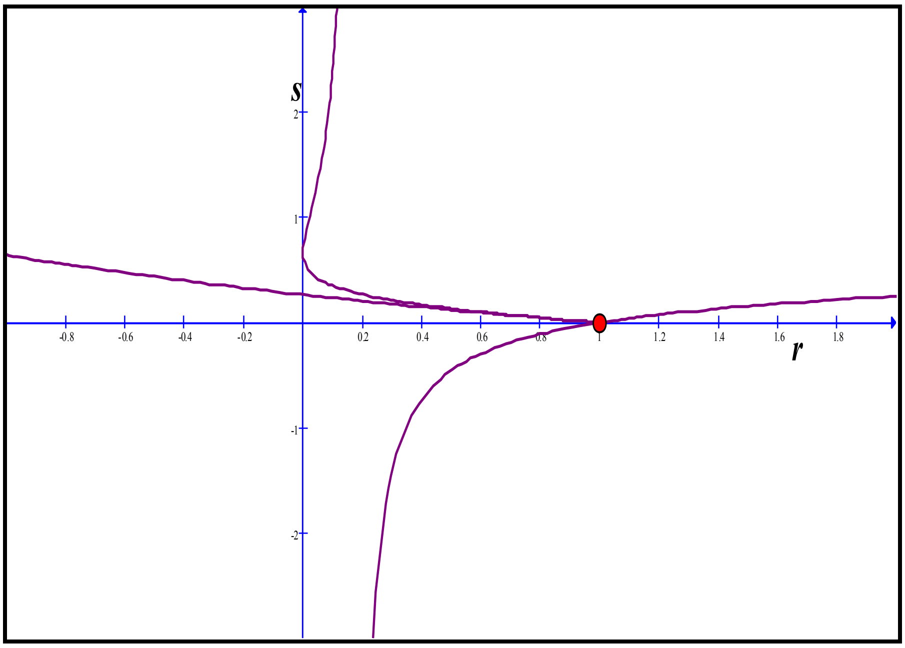

Fig. 5:-The statefinder parameter r v/s s

6. Conclusion

In the present paper, we have studied the plane symmetric cosmological model filled with interacting Dark matter and Holographic dark energy in Lyra geometry.To get the solutions of the field equations, we consider the scalar factor of the form $a = \frac{-1}{t} + t^2$, $t>1$. The decelerating parameter (q) is dynamical and attain value -0.5, shows that model is accelerating.The model is isotropy for initial time and then converted to anisotropy as the anisotropy parameter tends to infinity. It is found that EoS parameter of holographic dark energy starts from phantom region and remains in quintessence region for late time. It is also seen that, Lyra geometry is physically meaningful for finite time, but for very large time it is converted to general relativity.The statefinder parameter $\{r, s\}$ has calculated in order to differentiate our model with all other models of DE.

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