Gurukul International Multidisciplinary Research Journal (GIMRJ)*with* International Impact Factor 8.357
Peer Reviewed Journal
https://doi.org/10.69758/GIMRJ/2504I5VXIIIP0047

e-ISSN No. 2394-8426
Monthly Issue
APR-2025
Issue–IV, Volume–XIII

FRACTAL LIGHT FROM LASERS

Supriya Behere
Department of Physics, Shri Shivaji Arts, Commerce and Science College, Motala, Dist: Buldhana, (Maharashtra) Pin 443103
Email: shantipriyabehere@gmail.com

Abstract

Fractals are irregular shapes having self-similarity and scale invariance and have non-integral dimension. A concept of fractal theory and fractal geometry was put forward by Benoit B. Mandelbrot in early 1980’s, for the study of irregular shapes or non-Euclidean shapes. In nature we observe many irregular patterns. Snowflakes, clouds, mountains, coastlines, trees, lightning etc. which are examples of natural fractals.The standard mathematical fractal shapes can be described by: Koch Curve, Sierpinski Triangle famous Fractal Mandlebrot set,Julia set etc.

Fractal dimension of an object can be non-integral number in contrast to Euclidian geometry. A non-integral dimension such as D = 1.2 or 1.4 indicates that the object is having dimensions between 1 and 2.

It is the characteristic of a fractal object in which complete structure is comprised of its smallest part. Hence when each piece of a shape is like the whole, it is said to be self-similar.

The perfect fractal which possess symmetry over all orders of magnitude are called regular fractals or perfect fractals. Geometrical figures ore examples of these. But Fractals found in nature obliviously exhibit scaling over a limited range. There are various methods to calculate the fractal dimensions.

Recently the fractalsare also observed in laser beams.

Introduction

Mandelbrot coined the term ‘fractal’ from the Latin adjective ‘fractus’ mean "irregular," The term fractal (from the Latin *fractus*– irregular).Study of fractals started in early 1970s, their spectacular beauty fascinated many, to a layman and a professional alike,B. Mandelbrot [1] introduced the term fractal geometryin the middle of the seventies.Possibly nobody might have suspected the revolutionary effects of this new perspective on the sciences which arenowadays applied to natural and to mathematical effects. The classical Euclidean geometry that one learns in school is quite different than the fractal geometry, mostly because fractal geometry concerns non-linear, non-integral systems whereas Euclidean geometry is mainly oriented around linear, integral systems. Hence, Euclidean geometry is explanation of lines, ellipses, circles, or in general the regular shapes etc. However, fractal geometry is a description of algorithms to produce irregular shapes. Fractals are irregular shapes having self-similarity and scale invariance and have non-integral dimension. In nature we observe many irregular patterns. Snowflakes, clouds, mountains, coastlines, trees, lightning etc. are examples of natural fractals. It is the characteristic of a fractal object in which complete structure is comprised of its smallest part. Hence when each piece of a shape is like the whole, it is said to be self-similar.

The perfect fractal which possess symmetry over all orders of magnitude are called regular fractals or perfect fractals. Geometrical figures ore examples of these. But Fractals found in nature obliviously exhibit scaling over a limited range. There are various methods to calculate the fractal dimensions Fractal dimension of an object can be non-integral number in contrast to Euclidian geometry. A non-integral dimension such as D = 1.2 or 1.4 indicates that the object is having dimensions between 1 and 2. Recently the fractals are also observed in laser beams.

Fractals and Self-similarity

Gurukul International Multidisciplinary Research Journal (GIMRJ)*with*
International Impact Factor 8.357
Peer Reviewed Journal
https://doi.org/10.69758/GIMRJ/2504I5VXIIIP0047
e-ISSN No. 2394-8426
Monthly Issue
APR-2025
Issue–IV, Volume–XIII

The fundamental idea in the fractal geometry is to understand self-similarity[2]. It is the characteristic of a fractal object in which complete structure is comprised of its smallest part. Hence when each piece of a shape is similar to the whole, it is said to be self-similar. The presence of self-similarity in the object characterizes them as fractals, for example The Serpianski gasket, Kosch curve, Mandelbrot set etc

Koch Curve

To put up Koch Curve consider a straight line of unit length as shown in Fig1a. If this line is divided in three parts with the middle third of this line as base, construct an equilateral triangle and remove the base to get stage 1 as shown in Fig. 1b. The structure in this stage is known as "David's star". This Figure is made up of four segment of length 1/3 each so that total length is 4/3 units. Nowtake middle third of each segment as base construct equilateral triangles and remove the bases to get stage 2 as shown in Fig.1c. This Figure is made up of $16(4^2)$ segments of length $1/ 3^2$ each so that the total length is$16/3^2$ $[(4/3)^2]$units. Repeating this process, we get the Koch curveFig 1d

The Sierpinski Gasket

To make Sierpiniski Gasket initiate with an equilateral triangle of unit side length. Join Middle point of three Side lengths. By joining the middle point of three sides, we get 4 triangles. Remove the central triangle. the figure now consists of $3 = 3^1$ equilateral triangles of sides length $1/2 = 2^{-1}$ Now in each case of 3 equilateral triangles, again the central triangle obtained by joining the middle point of those small triangles. At this stage this figure consists of 3^2 equilateral triangles of side length $(½)/2 = ¼ = 1/ 2^2 = 2^{-2}$. As the process is continued, at the n^{th} stage. We have a figure consisting of 3^n equilateral triangles of side length 2^{-n}. The limiting figure of this process is called Sierpinski Gasket. Fig 2

The Mandelbrot set.

The Mandelbrot set is widely acknowledged fractal. Mandelbrot`s discovery is outcome of his research in the area of iteration theory, also known as complex analytic dynamics. This field dates back to the investigation of Fatou and Julia in the early part of this century. A one to one correspondence between the complex number and the points in the complex plane. Repeated application of a simple function causes some of these points to run away towards infinity, while others never wander far from the origin. The Mandelbrot set is made up of connected points in the complex plane. the simple equation, which is the basis of Mandelbrot, set is,

Changing number + Fixed number = Result. …….. (1)

In order to calculate points for a Mandelbrot set, let us start with one of the numbers on the complex plane and put its value as "fixed number" in equation

(1)The "changing number is repeated or iterated. This operation is repeated for infinite number of times. When iterative equations are applied to points in a certain region of complex plane, a Fractal from Mandelbrot set results. A fractal obtained from Mandelbrot set isshown in Fig.3

Fractal Dimension

There are various methods of calculating the fractal dimension from the diagram of a given shape. UsuallyBox-Counting (Grid) Method is employed. The image selected is to be digitized first and is converted into BMP image and then applied to box counting program. Then grids of various sizes are put on that map. The number of squares which are occupied by the squares are counted. This procedure is repeated for various sizes of the squares in the grid. For different values of size of the box 'r' the number of boxes N is found.Log(N) versus log(r) plot gives a straight line whose slope gives a fractal dimension. Points on the line are the least square fits to the data.

Applications of fractals

Gurukul International Multidisciplinary Research Journal (GIMRJ)*with*
International Impact Factor 8.357
Peer Reviewed Journal
https://doi.org/10.69758/GIMRJ/2504I5VXIIIP0047

e-ISSN No. 2394-8426
Monthly Issue
APR-2025
Issue–IV, Volume–XIII

There are large number of applications of fractals which range from weather patterns, share market, diffusion limited aggregation, shapes of lakes and water bodies photographed by google earth maps, electrodeposition ecology etc.[4 -6]An application of fractal geometry to lakes was studied by Shaikh Yusuf etal [7] So also the fluctuations in weather parameters were also studied by shaikh Yusuf etal[8]. One interesting application was studied [9] on the web knitting property of spider after administering different drugs like caffeine, marijuana, speed and sleeping pills. The photographs of webs after the drug was given to spider served the starting point from which the fractals dimensions were calculated.It was concluded that the sleeping pills and the caffeine affect adversely on the knitting ability of the spider. Comparatively marijuana and speed don't affect to that extent.

Fractals observed in laser beam

Fractals were observed in the light beam of a laser cavity where the beam is oscillating back and forth and bouncing between the mirrors at each pass. The mirrors can be aligned to image the light itself on each pass at a common image plane. In case where the returning light is smaller by a factor M, after one trip it will be smaller by M^2, after two trips by M^3 and so on. Each time the light returns to image plane in its smaller or larger versions thus showing a pattern in pattern or fractal. To see the fractal pattern, one must look inside the box and not in the beam coming out of the box. This was difficult to experiment with and therefore the experimental verification of the proposed theory in 1998 eluded till 2019.With their highly polished spherical mirrors, laser resonators are almost the precise opposite of Nature, and so it came as a surprise when, in 1998, transverse intensity cross-sections of the eigenmodes of unstable canonical resonators were predicted to be fractals (Fig 4) by Karman et al [10].The research was reported by the combined team from the university Witwatersrand (Wits) Johannesburg South Africa and the Glasgow university. The initial version of the experiment was built by Dr Darryl Naidoo and completed by HendSroor (Wits) [11].The theoretical expertise was provided by the Glasgow team and the experimental validation was made South African team at Wits.

Stages of construction of Koch curve

Fig. 1 a stage 1

Fig. 1 b stage 2

Fig. 1c stage 3

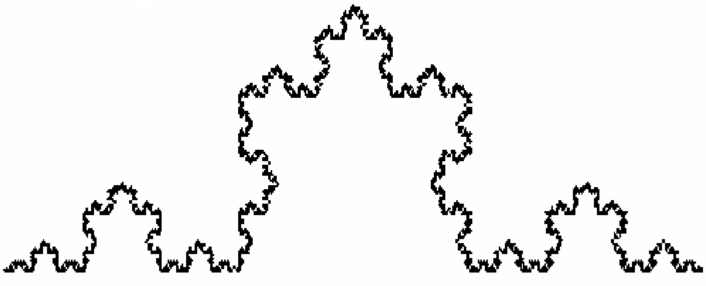
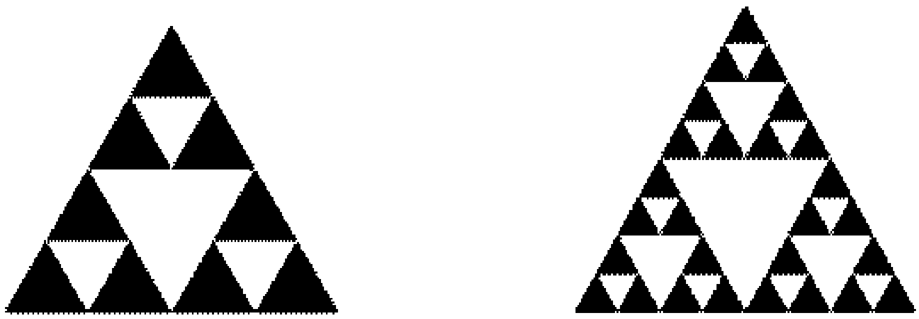


Fig 2Sierpiniski Triangle



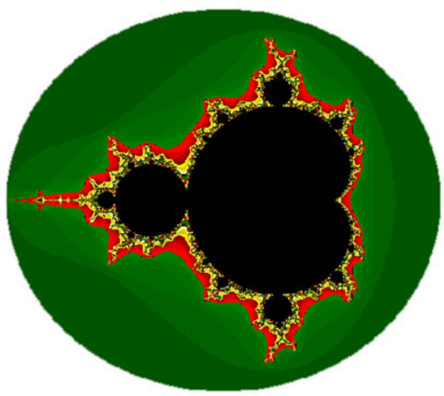


Fig. 3 Mandelbrot Set

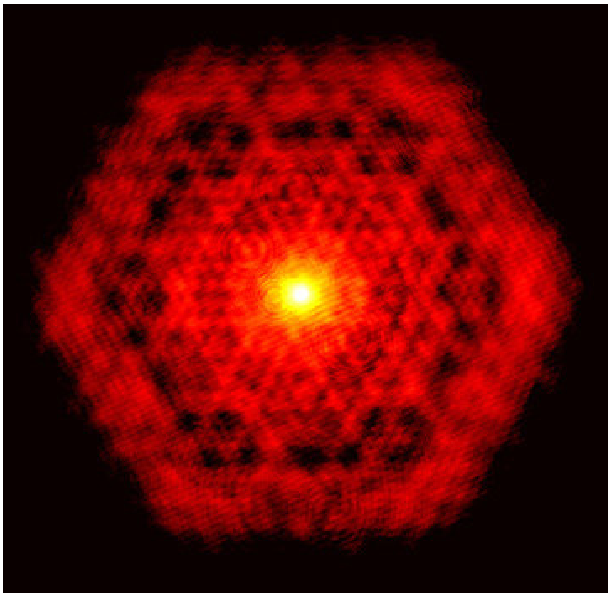


Fig 4. A fractal mode from a laser.

References

[1] Mandelbrot B.B., '*The Fractal geometry of nature'* Freeman, San Francisco
[2] Benoit B. Mandelbrot, 'How long is the Coast of Britain? Statistical Self similarity and Fractional Dimension,' *Science*, 637 (1967).
[3]]Hutchinson,J., *Fractals and self similarity*, Indiana university, journal of Mathematics **30,** 713, (1981).
[4] Harold M. H., Sugihra G., '*Fractals: A User's Guide for the Natural Sciences'* Oxford University, New York. (1993).
[5]Donald L. Turcotte, '*Fractal and chaos in geology and geophysics*' CambridgeUniversity press, (1992).
[6] Sugihara G and May R M, *Applications of fractals in ecology Trends Ecol. Evol.* **5**, 79, (1990).
[7]Shaikh Yusuf H, Pathan J M, Farooqui M, Khan A R and Behere S H,*Archives of Physics Research*, 1, 147 (2010)
[8]ShaikhYusuf H,Petagna K B . KhanA R, Pathan J M and. Behere S H, *Nonlinear Sci. Lett. B*, 1, 95 (2011)
[9] Chisty S Q and Supriya Behere , *Journal of the Gujrath research society,* 21, 15, 29-31, (2019)
[10]Karman G SMcDonald, G. H. C. New and J. P. Woerdman, *Nature, 402, 138; (1999)*
[11]*Sroor H*, Naidoo D, Miller S, Nelson J, Courtial J and Forbes A, *Phys. Rev. A 99, 013848.*(2019).