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TAKABAYASISTRING COSMOLOGICAL MODEL IN $f(R)$THEORY OF GRAVITY

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Abstract: The LRS Bianchi type-I cosmological model have been investigated in the context of $f(R)$ theory of gravity, focusing on the specific scenario involving cosmic stringsspecially on Takabayasi string. The solution of the field equations have been obtained by using the volumetric exponential law, the power law association between $f(R)$ and the average scale factor a. The physics and kinematical parameters of the model have been obtained and represented graphically.
Keywords: LRS Bianchi type-I metric; $f(R)$ gravity; modified gravity;Takabayasi strings.

1. INTRODUCTION

The early stages of the universe's evolution continue to be actively studied. After the Big Bang, the universe likely underwent several phase changes as it cooled. During these transitions, the symmetry breaking in particle physics resulted in the formation of topological imperfections like domain walls, strings, and monopoles [1]. Of the three topological defects discussed, only cosmic strings have the capability to produce particularly fascinating cosmological effects[2].

In recent years, numerous researchers have been motivated to explore cosmological models involving cosmic strings within the context of general theory of relativity, alternative gravitational theories and modified theories of gravitation. Katore and Hatkar [3] investigated both interacting and non-interacting situations involving two fluids within the framework of FRW space-time in the $f(R)$ theory of gravity. Capozzielloet *al.* [4] investigated the gravitational action at the tree level within bosonic string theory, considering its interaction with the dilaton field. Myung and Kim [5] studied Hoˇrava-Lifshitz black hole solutions and its thermodynamic properties. Mann found a class of black hole solutions to a $(3 + 1)$ dimensional theory gravity coupled to abelian gauge fields with negative cosmological constant that has been proposed as a dual theory to a Lifshitz theory describing critical phenomena in $(2 + 1)$ dimensions [6]. Santosh *et al.* [7] obtained exact solutions for isotropic homogeneous cosmological model with bulk viscosity. Yadav*et al.* [8] have studied some Bianchi type – I Viscous fluid string cosmological models with magnetic field. Pradhan and Rai [9] investigated the role of cosmic strings in Bianchi type-III spacetime in the presence of a bulk viscous fluid.

Wang [10] discussed Kantowski – Sachs string cosmological model with Bulk Viscosity in General Relativity.

Given the recent identification of the universe's accelerated expansion [11, 12], cosmologists are increasingly focusing on modified theories of gravity. These theories are gaining attention due to their potential to explain the acceleration of the universe. Notable examples include the $f(R, T)$ gravity theory [13] and $f(R)$ gravity [14, 15].

Bianchi type models, which exhibit homogeneity but may lack isotropy, seem to provide the most compelling framework for understanding possible anisotropic effects in the early universe.

Jaffe *et al.* [16] analyzed the impact of removing a Bianchi component from Wilkinson Microwave Anisotropy Probe (WMAP) data and found that this adjustment could account for several large-angle anomalies, ultimately supporting the notion of an isotropic universe. As a result, models incorporating an anisotropic background are considered particularly suitable for describing the universe's early stages. Among these, the Bianchi type-I model is one of the most straightforward representations of anisotropic backgrounds. In the context of f(R) gravity theory, many researchers have recently investigated various cosmological models based on anisotropic Bianchi types. Shamir [17] examined specific vacuum solutions for Bianchi type-I, III, and Kantowski–Sachs space-times within the metric formulation of $f(R)$ gravity. Several researchers devote their efforts towards the work in this context [18-20].

With the reference of above discussion, here we explored, LRS Bianchi type-ITakabayasistring cosmological model in the framework of $f(R)$ theory of gravity. The paper is organized as follows: In Sec. 2, we discussed the formation of $f(R)$ field equations for the LRS Bianchi type-I space-time. In section 3 the solution of field equation is given. In section 4 properties of model are discussed with graphical representation and in the last section 4 discussion and conclusions are given.

BASIC FORMATION.

The field equations of $f(R)$ gravity are obtained from the action

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G}f(R) + L_m\right)d^4x, \quad (1)$$

where $f(R)$ is a general function of the Ricci scalar and L_m is the matter Lagrangian. Variation of action (1) with respect to metric gives the following field equations:

$$F(R)R_{ij} - \frac{1}{2}f(R)g_{ij} - \nabla_i\nabla_j F(R) + g_{ij}\delta F(R) = kT_{ij} \quad (2)$$

where$F(R) = \frac{df}{dR}$ and $\delta = \nabla^i\nabla_i$, $\nabla_i$is the covariant derivative. Contracting the filed equation (2), we get

$$F(R)R - 2f(R) + 3\delta F(R) = kT\ , \quad (3)$$

Using above equation in Eq. (2), the field equations take the form

$$F(R)R_{ij} - \nabla_i\nabla_j F(R) - kT_{ij} = g_{ij}\left(\frac{F(R)R-\delta F(R)-kT}{4}\right), \quad (4)$$

Equation (3) is an important relationship between $f(R)$and$F(R)$ which will be used to simplify the field equations and to evaluate $f(R)$.

We consider the anisotropic LRS Bianchi type-I metric in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2), \quad (5)$$

where$A$ and B are functions of cosmic time t only.

The energy–momentum tensor for cosmic string source is given by

$$T_{ij} = \rho u_i u_j - \lambda a_i a_j, \quad (6)$$

whereλ is the string tension density, ρ is the rest energy density of the system and functions of time t only. Also, u_i is the four velocity vector, a_i is a space-like vector which represents the anisotropic directions of the string and they satisfy

$$g^{ij}u_i u_j = -a^i a_j = 1\ , \qquad u^i a_i = 0, \quad (7)$$

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We assume the string to be lying along the x-axis. The one-dimensional strings are assumed to be loaded with particle and energy density is $\rho_p = \rho - \lambda$. Latelier [18] has pointed out that λ may be positive or negative.

By adopting comoving coordinates, the field equations (4) for the metric (5) yield the following equations:

$$\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{F}\dot{B}}{FB} + \frac{\ddot{F}}{F} - \frac{f(R)}{2F} = \frac{k\,\lambda}{F} \tag{8}$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \frac{\dot{F}}{F}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) - \frac{f(R)}{2F} = \frac{k\rho}{F} \tag{9}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{\dot{F}}{F}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\ddot{F}}{F} - \frac{f(R)}{2F} = 0 \tag{10}$$

where overhead dot stands for ordinary differentiation with respect to cosmic time t.

We define the following some important physical and kinematical parameters for the model (5).

Hubble's parameter of the model

$$H = \frac{\dot{r}}{r} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{11}$$

where$r = (AB^2)^{\frac{1}{3}}$ is average scale factor.

The volume of model is defined as

$$V = (AB^2)^3 \quad , \tag{12}$$

Expansion scalar and shear scalar are defined

$$\theta = u^i_{;i} = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \quad , \tag{13}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \quad , \tag{14}$$

whereσ_{ij} is shear tensor.

Anisotropic parameter Δ is given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \quad , \tag{15}$$

where$H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$are directional Hubble's parameter, which express the expansion rates of the universe in the directions of x, y and z respectively.

Deceleration parameter is given by

$$q = -\frac{\ddot{r}r}{\dot{r}^2} \quad , \tag{16}$$

The behavior of the universe models is determined by the sign of the q. The positive value of q suggests deceleration model while the negative value indicates inflation.

The scalar curvature for the metric (5) is given by

$$R = 2\left(\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB}\right), \tag{17}$$

2. SOLUTION OF FILED EQUATIONS

Now the field equations (8) - (10) reduces to the following independent equations

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{F}}{F}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = \frac{k\,\lambda}{F}, \tag{18}$$

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$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}-\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{F}}{F}\frac{\dot{B}}{B}-\frac{\ddot{F}}{F}=\frac{k\rho}{F}, \tag{19}$$

The equations (18) and (19) are two independent equations with five unknown $A, B, f(R), \rho$ and λ. Hence to find a determinate solution we use the following physically possible conditions:

(I) We Consider the volumetric exponential law given by

$$V = p_1 e^{3tn}, \tag{20}$$

here p_1 and n are constants.

(II) The power law relation between F and average scale factor $a(t)$ is [20]

$$f(R) = F_0 a^n, \tag{21}$$

here we take $F_0 = 1$ is proportionally constant and n is any integer.

(III) The equation of state for string model is [21]

$$\rho = \varpi\lambda \quad , \tag{22}$$

where the constant $\varpi = 1 + \omega$ defines Takabayasi string.

3. Properties of Takabayasi String cosmological model ($\rho = (1 + \omega)\lambda$).

Using equations (18)- (21), we get the metric potentials as

$$A = p_1 e^{\frac{3tn^2+2e^{2tn}p_1{}^2Q_1}{3n}}, \tag{23}$$

$$B = p_1^{1/3} e^{tn-\frac{e^{3tn}p_1Q_1}{9n}}, \tag{24}$$

The equation (5) can be written as

$$ds^2 = dt^2 - \left[p_1 e^{\frac{3tn^2+2e^{2tn}p_1{}^2Q_1}{3n}}\right]^2 dx^2 - \left[p_1^{\frac{1}{3}} e^{tn-\frac{e^{3tn}p_1Q_1}{9n}}\right]^2 (dy^2 + dz^2), \tag{25}$$

where P_1 and Q_1 are constant of integration.

The volume of the model is given by

$$V = p_1 e^{3nt}, \tag{26}$$

here p_1 and n are constants.

The directional Hubble parameters are given by

$$H_x = \frac{n+2e^{tn}}{3} p_1 Q_1, \tag{27}$$

$$H_y = \frac{n-e^{tn}}{3} p_1 Q_1, \tag{28}$$

The mean Hubble parameter is given by

$$H = \frac{n}{3}, \tag{29}$$

The anisotropic parameter is given by

$$\Delta = \frac{2e^{2nt}p_1^2Q_1^2}{3n^{2^2}}, \tag{30}$$

The expansion scalar is given by

$$\theta = n, \tag{31}$$

The shear scalar is given by

$$\sigma^2 = \frac{2ne^{2nt}p_1^2Q_1^2}{9n^{2^2}}, \tag{32}$$

The Ricci scalar R is given by

$$R = \frac{2}{3}(2n^2 + e^{2nt}p_1^2Q_1^2), \tag{33}$$

The function $f(R)$ is

$$f(R) = \frac{4n^2p_1^2Q_1^2e^{t\left(\frac{k}{n}+2n\right)}}{3(k+2n^2)}, \tag{34}$$

The energy density is given by

$$\rho = \frac{e^{\frac{tk}{n}}(-9k^2 + 3kn^2 - 2n^4 + np_1Q_1e^{nt}(-3k + 10n^2 + np_1Q_1e^{nt}))}{9n^2\kappa}$$

$$\lambda = (1+\omega)\frac{e^{\frac{tk}{n}}(-9k^2 + 3kn^2 - 2n^4 + np_1Q_1e^{nt}(-3k + 10n^2 + np_1Q_1e^{nt}))}{9n^2\kappa}$$

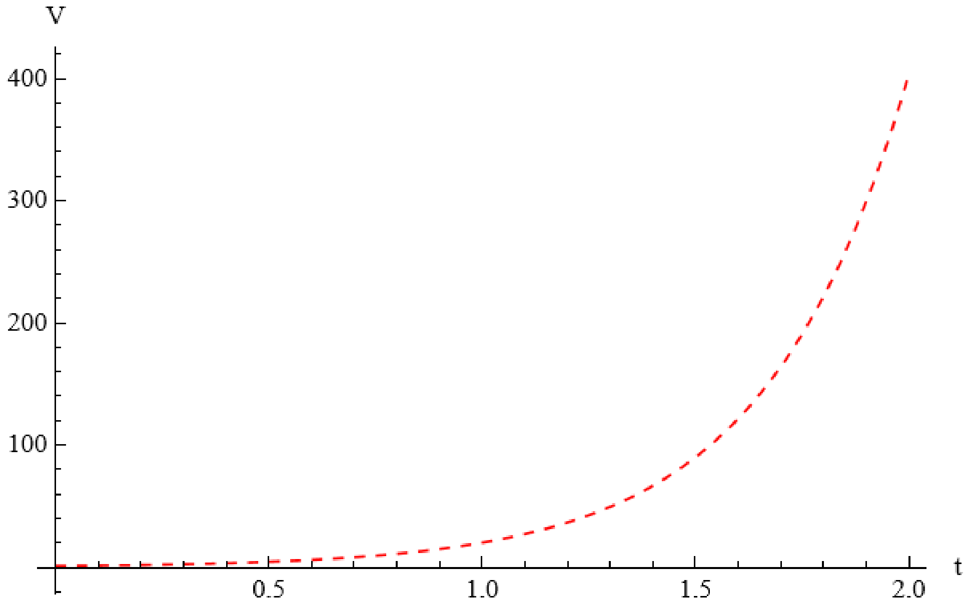

Fig-1: Volume verses time plotted for $n = -2, k = 5, Q_1 = 1, p_1 = 1$.

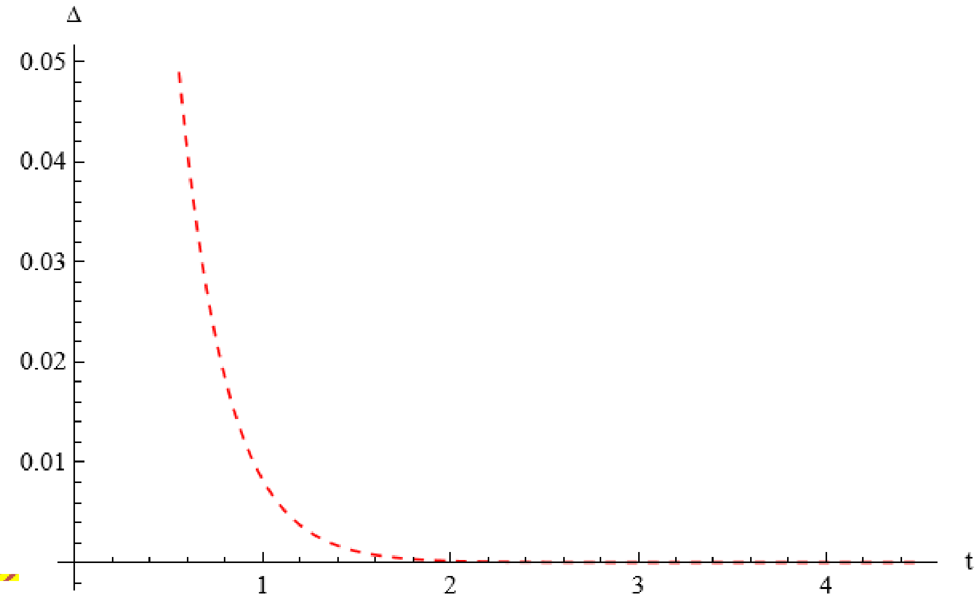

Fig-2: Anisotropic parameter verses time plotted for $n = -2, k = 5, Q_1 = 1, p_1 = 1$.

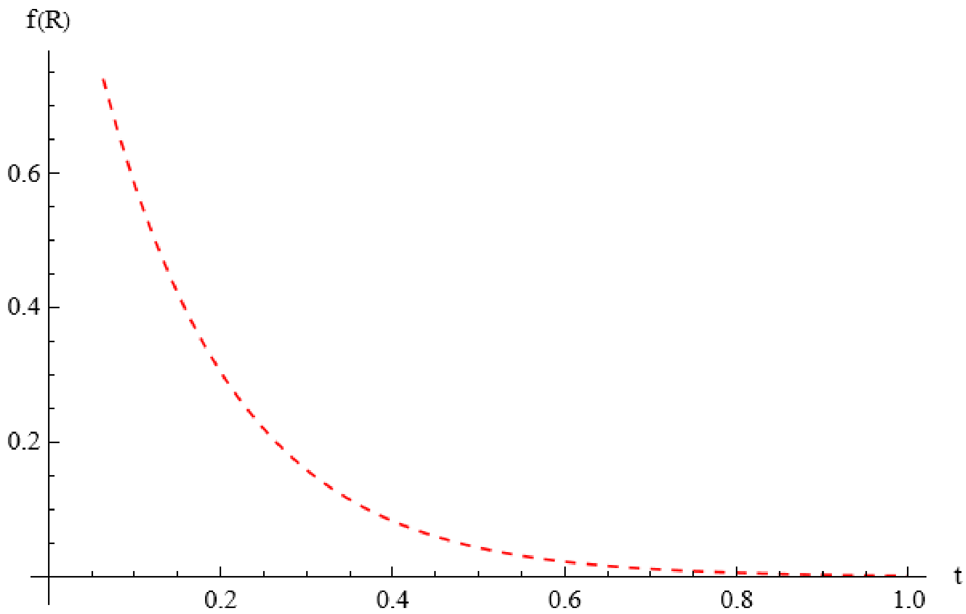

Fig-3: $f(R)$ verses time plotted for $n = -2, k = 5, Q_1 = 1, p_1 = 1$.

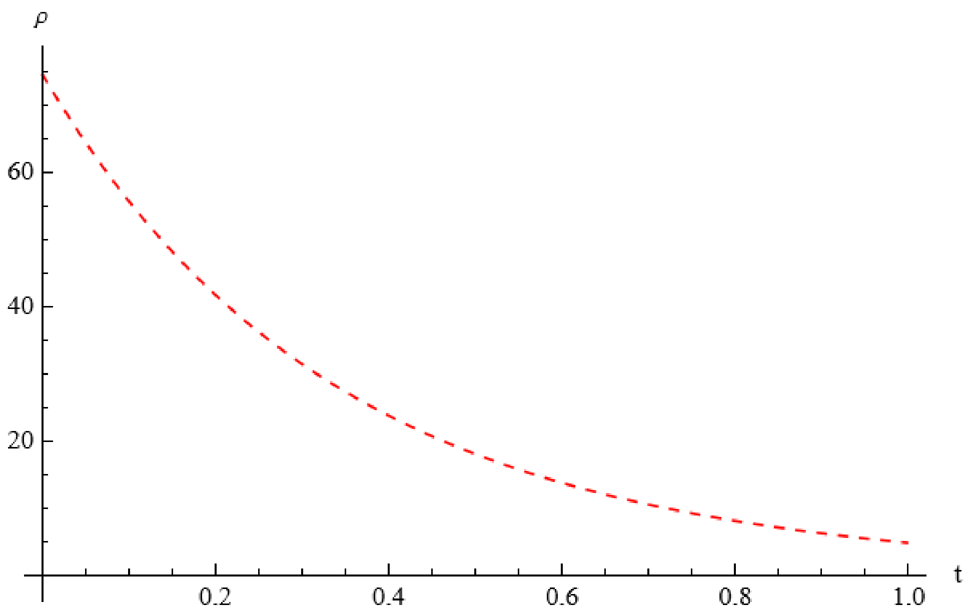

Fig-4: Energy density verses time plotted for $n = -2, k = 5, Q_1 = 1, p_1 = 1$, $\kappa = -0.1$

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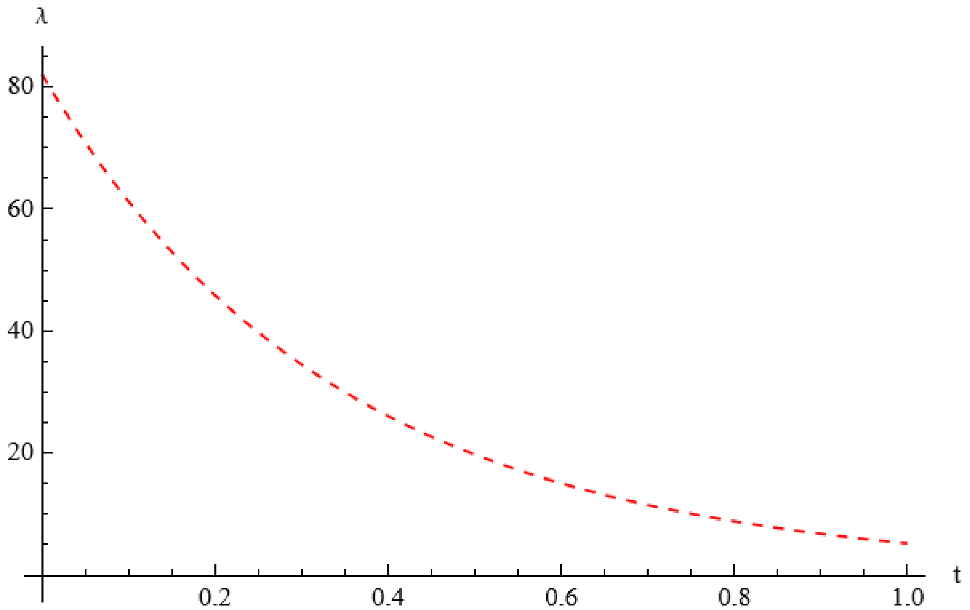

Fig-5: Energy density verses time plotted for$n = -2, k = 5, Q_1 = 1, p_1 = 1$, $\kappa = -0.1$

4. DISCUSSION AND CONCLUSION

In the nutshell, we have investigated LRS Bianchi type-I Takabayasistring cosmological models in $f(R)$ theory of gravitation. The physical behavior of this model can be summarized as follows. Form the figure -1 it is clear that the universe's spatial volume is growing exponentially as time progresses in cosmic time$t$. From figure-2, it is clear that the anisotropic parameter decreases over cosmic time tt and eventually reaches zero at a certain stage. Hence it shows anisotropic nature after some finite time$t$. Figure-3, represents the graph of function of Ricci scalar with respective the cosmic time t, depicts the decreasing nature. From figure-4 and figure-5, the energy density and the string tension density of the universe is decreasing with respective the cosmic time t. This shows that the universe moving towards the vacuum and hence it is expanding. Hence all the observations are allied with the recent observations of the universe.

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